TGD and M-Theory

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Abstract

In this chapter a critical comparison of M-theory and TGD as two competing theories is carried out. Dualities and black hole physics are regarded as basic victories of M-theory.

a) The counterpart of electric magnetic duality plays an important role also in TGD and it has become clear that it might change the sign of Kähler coupling strength rather than leaving it invariant. The different signs would be related to different time orientations of the space-time sheets. This option is favored also by TGD inspired cosmology but unitarity seems to exclude it.

b) The AdS/CFT duality of Maldacena involved with the quantum gravitational holography has a direct counterpart in TGD with 3-dimensional causal determinants serving as holograms so that the construction of absolute minima of Kähler action reduces to a local problem.

c) The attempts to develop further the nebulous idea about space-time surfaces as quaternionic sub-manifolds of an octonionic imbedding space led to the realization of duality which could be called number theoretical spontaneous compactification. Space-time can be regarded equivalently as a hyper-quaternionic 4-surface in M^8 with hyper-octonionic structure or as a 4-surface in $M^4 \times CP_2$.

d) The duality of string models relating Kaluza-Klein quantum numbers with YM quantum numbers could generalize to a duality between 7-dimensional light like causal determinants of the imbedding space (analogs of "big bang") and 3-dimensional light like causal determinants of space-time surface (analogs of black hole horizons).

e) The notion of cotangent bundle of configuration space of 3-surfaces suggests the interpretation of the number-theoretical compactification as a wave-particle duality in infinite-dimensional context. Also the duality of hyper-quaternionic and co-hyper-quaternionic 4-surfaces could be understood analogously. These ideas generalize at the formal level also to the M-theory assuming that stringy configuration space is introduced. The existence of Kähler metric very probably does not allow dynamical target space.

In TGD framework black holes are possible but putting black holes and particles in the same basket seems to be mixing of apples with oranges. The role of black hole horizons is taken in TGD by 3-D light like causal determinants, which are much more general objects. Black hole-elementary particle correspondence and p-adic length scale hypothesis have already earlier led to a formula for the entropy associated with elementary particle horizon.

The recent findings from RHIC have led to the realization that TGD predicts black hole like objects in all length scales. They are identifiable as highly tangled magnetic flux tubes in Hagedorn temperature and containing conformally confined matter with a large Planck constant and behaving like dark matter in a macroscopic quantum phase. The fact that string like structures in macroscopic quantum states are ideal for topological quantum computation modifies dramatically the traditional view about black holes as information destroyers.

The discussion of the basic weaknesses of M-theory is motivated by the fact that the few predictions of the theory are wrong which has led to the introduction of anthropic principle to save the theory. The mouse as a tailor history of M-theory, the lack of a precise problem to which M-theory would be a solution, the hard nosed reductionism, and the censorship in Los Alamos archives preventing the interaction with competing theories could be seen as the basic reasons for the recent blind alley in M-theory.

1 Introduction

In this chapter a critical comparison of M-theory [28] and TGD (see [1, 2, 4, 5, 3, 6, 7] and [10, 8, 9, 13, 11, 12, 14, 15]) as two competing theories is carried out. Also some comments about the sociology of Big Science are made.

1.1 From hadronic string model to M-theory

The evolution of string theories began 1968 from Veneziano formula realizing duality symmetry of hadronic interactions. It took two years to realize that Veneziano amplitude could be interpreted in terms of interacting strings: Nambu, Susskind and Nielsen made the discovery simultaneously 1970. The need to describe also fermions led to the discovery of super-symmetry [21] and Ramond and Neveu-Schwartz type superstrings in the beginning of seventies.

Gradually it became however clear that the strings do not describe hadrons: for instance, the critical dimensions for strings *resp.* superstrings where 26 *resp.* 10, and the breakthrough of QCD at 1973 meant an end for the era of hadronic string theory. 1974 Schwartz and Scherk proposed that strings might provide a quantum theory of gravitation [22] if one accepts that space-time has compactified dimensions.

The first superstring revolution was initiated around 1984 by the paper by Green and Schwartz demonstrating the cancellation of anomalies in certain superstring theories [23, 24]. The proposal was that superstrings might provide a divergence-free and anomaly-free quantum theory of gravitation. A crucial boost was given by Witten's interest on superstrings. Also the highly effective use of media played a key role in establishing superstring hegemony.

It became clear that superstrings come in five basic types [26]. There are type I strings (both open and closed) with $N = 1$ super-symmetry and gauge group $SO(32)$, type IIA and IIB closed strings with $N = 2$ super-symmetry, and heterotic strings, which are closed and possess $N = 1$ super-symmetry with gauge groups $SO(32)$ and $E^8 \times E^8$. There is an entire landscape of solutions associated with each superstring theory defined by the compactifications whose dynamics is partially determined by the vanishing of conformal anomalies. For a moment it was believed that it would be an easy task to find which of the superstrings would allow the compactification which corresponds to the observed Universe but it became clear that this was too much to hope. In particular, the number 4 for non-compact space-time dimensions is by no means in a special position.

Around 1995 came the second superstring revolution with the idea that various superstring species could be unified in terms of an 11-dimensional M-theory with M meaning membrane in the lowest approximation [28]. M-theory allowed to see various superstrings as limiting situations when 11-D theory reduces to 10-D one so that very special kind of membranes reduce to strings. This allowed to justify heuristically the claimed dualities between various superstrings [26]. Matrix Theory as a proposal for a non-perturbative formulation of M-theory appeared 2 years later [29].

Now, almost a decade later, M-theory is in a deep crisis: the few predictions that the theory can make are definitely wrong and even anthropic principle is advocated as a means to save the theory [32]. Despite this, very many people continue to work with M-theory and fill hep-th with highly speculative preprints proving that this is dual with that although the flow of papers dealing with strings and M-theory has reduced dramatically.

A reader interested in critical views about string theory can consult the article of Smolin [33] criticizing anthropic principle, the web-lectures "Fantasy, Fashion, and Faith in Theoretical Physics" of Penrose [30] as well as his article in New Scientist [31] criticizing the notion of hidden space time dimensions, and the articles of Peter Woit [34]. Also the discussion group "Not Even Wrong" [35] gives a critical perspective to the situation almost a decade after the birth of M-theory.

1.2 Evolution of TGD briefly

The first superstring revolution shattered the world at 1984, about two years after my own doctoral dissertation (1982), and four years after the Esalem conference in which the quantum consciousness movement started. Remarkably, David Finkelstein was one of the organizers of the conference besides being the chief editor of "International Journal of Theoretical Physics", in which I managed to publish first articles about TGD. The first and last contact with stars was Wheeler's review of my first article published in IJTP, and I cannot tell what my and TGD's fate had been without Wheeler's highly encouraging review.

During the 27 years after the discovery that space-times could be regarded as 4-surfaces as well as extended objects generalizing strings, I have devoted my time to the development of TGD. Without exaggeration I can say that life devoted to TGD has been much more successful project than I dared or even could dream and has led outside the very narrow realms of particle physics and quantum gravity. Indeed, without knowing anything about Finkelstein and Esalem at that time, I started to write a book about consciousness around 1995 when the second superstring revolution occurred. TGD inspired theory of consciousness has now materialized as 8 online books at my home page.

Altogether these 27 years boil down to seven online books [1, 2, 4, 5, 3, 6, 7] about TGD proper and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15] plus printed book about TGD [TGD]. This makes about 8000 pages of TGD spanning everything between elementary particle physics and cosmology. One might expect that the sheer waste amount of material at my web site might have stirred some interest in the physics community despite the fact that it became impossible to publish anything and to get anything into Los Alamos archives after the second super-string revolution. The only visible reaction has been from my Finnish colleagues and guarantees that I will remain unemployed in the foreseeable future. I will discuss some reasons for this state of affairs after comparing string models and TGD, and considering the reasons for the failure of the theory formerly known as superstring model.

Before continuing, I hasten to admit that I am not a string specialist and I do not handle the technicalities of M-theory. On the other hand, TGD has given quite a good perspective about the real problems of TOEs and provides also solutions to them. Hence it is relatively easy to identify the heuristic and usually slippery parts of various arguments from the formula jungle. Also I want to express my deep admiration for the people living in the theory world but from my own experience I know how easy it is to fall on wishful thinking and how necessary but painful it is to lose face now and then.

My humble suggestion is that M-theorists might gain a lot by asking what "What possibly went wrong?". This chapter suggests answers to this question. Perhaps M-theorists might also spend few hours in the web to check whether M-theory is indeed the only viable approach to quantum gravity: the material at my own home page might provide a surprise in this respect.

2 A summary about the evolution of TGD

The basic idea about space-time as a 4-surface popped in my mind in autumn at 1978. The first implication was that I lost my job at Helsinki University. During the next 4 years this idea led to a thesis with the title "Topological GeometroDynamics" (TGD), which I think was suggested by David Finkelstein to distinguish TGD from Wheeler's GeometroDynamics.

2.1 Space-times as 4-surfaces

TGD (for a summary about the evolution of TGD see [A1, A2]) can be seen as as a solution to the energy problem of General Relativity via the unification of special and general relativities by assuming that space-times are representable as 4-surfaces in certain 8-dimensional space-time with the symmetries of empty Minkowski space. An alternative interpretation is as a generalization of string models by replacing strings with 3-dimensional surfaces: depending on their size they would represent elementary particles or the space we live in and anything between these extremes. From this point of view superstring theories are unique candidates for a Theory of Everything if space-time were 2- rather than 4-dimensional.

The first superstring revolution made me happy since I was convinced that it would be a matter of few years before TGD would replace superstring models as a natural generalization allowing to understand the four-dimensionality of the space-time. After all, only a half-page argument, a simple exercise in the realization of standard model symmetries, leads to a unique identification of the higher-dimensional imbedding space as a Cartesian product of Minkowski space and complex projective space CP_2 unifying electro-weak and color symmetries in terms of its holonomy and isometry groups. By the 4-dimensionality of the basic objects there was no need for the imbedding space geometry to be dynamical. Theory realized the dream about the geometrization of fundamental interactions and predicted the observed quantum numbers. In particular, the horrors of spontaneous compactification to be crystallized in the notion of M-theory landscape two decades later can be circumvented completely.

2.2 Uniqueness of the imbedding space from the requirement of infinite-dimensional Kähler geometric existence

Later I discovered heuristic mathematical arguments suggesting but not proving that the choice of the imbedding space is unique. The arguments relied on the uniqueness of the infinite-dimensional Kähler geometry of the configuration space of 3-surfaces. This uniqueness was discovered already in the context of loop spaces by Dan Freed [20].

CH, the "world of the classical worlds" serves as the arena of quantum dynamics [B2, B3], which reduces to the theory of classical spinor fields in CH and geometrizes fermionic anti-commutation relations and the notion of super-symmetry in terms of the gamma matrices of CH [B4]. Only quantum jump is the genuinely non-classical element of the theory in CH context. The heuristic argument states that CH geometry exists only for $H = M^4 \times CP_2$.

In particular, number theoretical arguments relating to quaternions and octonions fix the dimensions of space-time and imbedding space to four and 8 respectively. The fact that the space of quaternionic sub-spaces of octonion space is CP_2 suggest an explanation for the special role of CP_2 [C5].

This stimulated a development, which led to notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 or as surfaces in $M^4 \times CP_2$ [E2]. What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multiplied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

The realization of 4-D general coordinate invariance forces to assume that Kähler function assigns a unique space-time surface to a given 3-surface: by the breakdown of the strict classical determinism of Kähler action unions of 3-surfaces with time like separations must be however allowed as 3-D causal determinants (CDs) and quantum classical correspondence allows to interpret them as representations of quantum jump sequences at space-time level. Space-time surface defined as absolute minimum or some more general preferred extremal [E2] of Kähler action is analogous to Bohr orbit so that classical physics becomes part of the definition of configuration space geometry rather than being a result of a stationary phase approximation.

2.3 The lift of 2-dimensional conformal invariance to the space-time level and field particle duality as the mother of almost all dualities

The basic argument against higher-dimensional extended objects is the loss of conformal invariance. This argument does not bite in TGD. The crucial conformal invariance of string models generalizes to the conformal invariance associated with two kinds of causal determinants (CDs) appearing in TGD and giving rise to quantum gravitational holography [B2, B3, B4]. Note that I ended up with the quantum gravitational holography in TGD sense already around 1990 and much before Maldacena discovered AdS/CFT correspondence [36].

The two types of light like CDs correspond to 3-D light-like surfaces $X^3 \subset X^4$ and 7-D light like surfaces of form $V_3 \times CP_2 \subset H$, V^3 a light like 3-surface of M^4. The metric 2-dimensionality of the light like 3-surface X^3 *resp.* V^3 implies super-conformal invariance realized at the level of space-time *resp.* imbedding space.

2.3.1 The particle aspect of the duality

The light like 3-surfaces $X_l^3 \subset X^4$ define CDs of first kind and correspond to a conformal invariance analogous to the world sheet conformal invariance of closed super string models. On the other hand, light like CDs are in a role analogous to the ends of open strings. The space-time surface itself relates to the light like CDs like the 3-dimensional space having 2-dimensional space-time as its boundary in WZW model. The modified Dirac equation at the light like CDs decouples from the normal coordinate and the solutions generalize the shock waves (light fronts) restricted to a 3-D surface in M^4. Thanks to the classical gravitation the light like 3-surface need not look expanding in H and can represent a particle at rest like black hole horizon. In particular, elementary particles and wormhole contacts are accompanied by what I call elementary particle horizons. The natural identification would be that the light like 3-D CDs represent the particle aspect: the interaction of the CD of photon with the detector makes it click just like a shock wave causes a bang in my ears. The solutions of the modified Dirac equation at light like CDs allow $N = 4$ local super-conformal invariance in both quark and lepton sector but it has become clear that there is no corresponding global super-symmetry (and no sparticles) and that super-conformal algebra is realized as dynamical symmetries at the configuration space level.

The strongest form of the quantum gravitational holography would be that the vertices needed to construct the S-matrix in quantum TGD are expressible in terms of the correlation functions of the $N = 4$ super-conformal field theory at the light like CD:s.

2.3.2 The field aspect of the duality

The light like 7-surfaces $\delta M^4_\pm \times CP_2$ appearing as the Cartesian factor of the boundary of $M^4_\pm \times CP_2$ define CDs of second kind. The positions for the dips of various light cones span $H = M^4 \times CP_2$ and unions of these CDs define sectors of the configuration space (note that Poincare invariance is exact). Note that one cannot yet exclude more general light like 7-surfaces $V_3 \times CP_2$ than light cones as CDs. Configuration space metric and spectrum generating algebra are constructed in terms of Hamiltonians in the symplectic algebra of $V^3 \times CP_2$ and spinor harmonics of the imbedding space. Hence everything is expressed in terms of imbedding space data.

The 3-surfaces X^3 restricted at 7-D CDs are space-like and the modified Dirac operator obviously does not allow shock wave solutions and solutions are genuinely four-dimensional. The solutions of Dirac equation represent gauge super-symmetries and also this symmetry can be regarded $N = 4$ super-conformal invariance [B4] and reduces to $N = 1$ global super symmetry coded into CP_2 geometry. The requirement that the net conserved quantum numbers are vanishing requires that a branching to positive and negative energy 4-surfaces occurs at X^3. The negative energy 3-surface from geometric future suffers time reflection as a positive energy surface: positive and negative energy matter is created from vacuum. The expressions of super charges and other spectrum generating charges involve contractions of imbedding space spinor harmonics with the second quantized induced spinor field at the space-like boundary components X^3 at 7-D causal determinants.

The are reasons to suspect that these CDs are in a dual relation in the sense that 7-D CDs provide a field description based on fields in the interior of X^4 and 3-D CDs provide a particle description in terms of "shock fronts". This 7-3 duality would be nothing but the age old field-particle duality but also the TGD counterpart for YM-gravitational and AdS/CFT dualities of string models. The most optimistic (from the point of view of calculability) guess is that by the classical non-determinism of Kähler action one can construct a space-time surface by fixing an arbitrary collection of either light like 3-D CDs or space-like 3-surfaces at 7-D CDs but not both. This dynamical cobordism made possible by the non-determinism of Kähler action at 3-D light like CDs could be regarded as a generalization of the topological cobordism and could be also non-trivial. TGD could be seen as a generalization of topological quantum field theories (TQFTs) [19, O3]. For instance, the orbits of particles represented by 3-D CDs could restricted within a space-time volume characterized by an appropriate p-adic length scale. This would allow to realize momentum and color eigen states.

2.4 TGD inspired theory of consciousness and other developments

During the last decade a lot has happened in TGD and it is sad that only those colleagues with mind open enough to make a visit my home page have had opportunity to be informed about this. Knowing the fact that a typical theoretical physicist reads only the articles published in respected journals about his own speciality, one can expect that the number of these physicists is not very high. Some examples of the work done during this decade are in order.

I have developed quantum TGD in a considerable detail with highly non-trivial number theoretical speculations relating to Riemann hypothesis and Riemann Zeta in general [E8, C5]. One outcome is a proposal for the proof of Riemann hypothesis [16]. Second outcome is a proposal for a general formula for the zeros explaining the observed translational invariance of the correlation functions for the zeros and required by purely physical constraints [C5].

During the same period I have constructed TGD inspired theory of consciousness [10]. One outcome is a theory of quantum measurement and of observer having direct implications for the quantum TGD itself. The results of the modification of the double slit experiment carried out by Afshar [49, 17] provides a difficult challenge for the existing interpretations of quantum theory and a support for the TGD view about quantum measurement in which space-time provides correlates for the non-deterministic process in question. The new views about energy and time have also profound technological implications.

TGD has forced the introduction of p-adic number fields besides real numbers and led to a generalization of number concept: p-adic number fields play a key role in the proposed physics of cognition and intentionality [E4, E6]. The notion of infinite primes [E3] leads to a generalization of the notion of space-time point [E10]. Space-time point becomes infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography. Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

p-Adic number fields lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [E4, E6]. One of the first applications was the calculation of elementary particle masses [F2, F3]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra is involved. Not only the fundamental mass scales reduce to number theory but also individual masses are predicted correctly under very mild assumptions. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [F3].

TGD inspired cosmology can be regarded as a fractal cosmology containing cosmologies within cosmologies [D5]. Sub-cosmology is defined in extremely general sense so that even the evolution of living organisms shares some crucial common aspects with cosmology in this sense. Initial singularities are absent. A period of flatness of 3-space following "big bang" is predicted by quantum criticality. The explanation of dark energy and dark matter are basically in terms of many-sheeted space-time although also new kinds of elementary particles are predicted (an entire hierarchy of asymptotically non-free standard model physics is possible). Dark matter and energy reside at larger space-time sheets, mainly magnetic flux quanta carrying magnetic and Z^0 magnetic fields. Solar corona represent a leakage of dark matter to our space-time sheets from magnetic flux tubes. Cosmological constant is predicted to have a spectrum given in terms of p-adic length scales characterizing the sizes of space-time sheets, and the deep puzzle produced by 10^{52}-fold discrepancy between experiment and theory disappears. Both the acceleration of cosmic expansion and the observed jerk [40] is understood.

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix. This has turned out to be the case to the extend that a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges. The theory leads also to a prediction for the spectrum of Planck constants associated with M^4 and CP_2 degrees of freedom.

2.5 Von Neumann algebras and TGD

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

2.5.1 How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

a) Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

b) Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

c) There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

d) Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of configuration space of 3-surfaces, etc....

2.5.2 Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

a) A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

b) At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

c) For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

d) The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

e) Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

2.6 Does dark matter at larger space-time sheets define superquantal phase?

The last step in the rapid evolution of quantum TGD [D6, 48] was stimulated when I learned that D. Da Rocha and Laurent Nottale [**?**] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.82 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The basic objection is that astrophysical systems are extremely classical whereas TGD predicts macrotemporal quantum coherence in the scale of life time of gravitational bound states. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

The earlier work with topological quantum computation [O3] had already led to the idea that Planck constant could depend on the quantum phase $q = exp(i\pi/n)$. The first attempts to understand the large values of the Planck constant led to a badly wrong formula for this dependence. The improved understanding of Jones inclusions and their role in TGD [C6] allowed to deduce an extremely simple formula for the Planck constant, as a matter fact, for the two separate Planck constants assignable to with M^4 and CP_2 degrees of freedom appearing as scaling factors of the corresponding metrics. These Planck constants are given by the formulas $\hbar(M^4) = n(CP_2)\hbar_0$ and $\hbar(CP_2) = n(M^4)\hbar_0$ in terms of integers defining the corresponding quantum phases. The far reaching implication is that Planck constants can have arbitrarily large values. In this framework even imbedding space is a concept emerging from infinite-dimensional Clifford algebra but only the scaling factors of the metric can vary.

The general philosophy would be that when the quantum system becomes non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character. This would apply to QCD and to atoms with $Z > 137$ and to any other system. $q \neq 1$ quantum groups characterize non-perturbative phases.

The values of n for which the quantum phase is expressible using only iterated square root operation (corresponding polygon is obtained by ruler and compass construction) are of special interest since they correspond to the lowest evolutionary levels for cognition so that corresponding systems should be especially abundant in the Universe. It should be noticed that this quantization does not depend at all on the parameter v_0 appearing in the formula of Nottale and this gives strong additional constraints to the ratios of planetary masses and also on the masses themselves if one assumes that the gravitational Planck constant corresponds to the values allowed by ruler and compass construction. Also correct prediction for the ratio of densities of visible and dark matter emerges.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The value of v_0 has interpretation as velocity of distant stars around galaxies in the gravitational field of long cosmic string like objects traversing through galactic plane. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. Sub-harmonics would result when cosmic strings decay to magnetic flux tubes: magnetic energy density per unit length is quantized by absolute minimization of Kähler action and the simplest possibility is the reduction of the energy density by a factor $1/n^2$.

v_0 can be expressed in terms of Kähler coupling strength α_K and the parameter R^2/G characterizing CP_2 size. The value $v_0 = 2^{-11}$ favored both by the planetary Bohr orbitology and quantum model for living matter leads to new insights about coupling constant evolution. The surprising find was that α_K is very nearly equal to the electro-weak coupling $\alpha_{U(1)}$. This observation led to new insights about coupling constant evolution.

a) Contrary to the earlier beliefs, it is possible to assume that α_K is renormalization group invariant in strong sense if one assumes that gravitational interactions are mediated by space-time sheets labelled by M_{127}, the largest Mersenne prime which does not correspond to super-astronomical length scale.

b) Since classical color action reduces to Kähler action as does also electro-weak U(1) action, and since color holonomy is Abelian and induced spinors fields carry only anomalous color hyper charge as spinlike color quantum number identical with electroweak hypercharge, one can argue that the sum of color and U(1) actions equals to Kähler action implying $1/\alpha_s + 1/\alpha_{U(1)} = 1/\alpha_K$ reducing the difficult-to-calculate evolution of color coupling strength to that of electroweak coupling constant evolution calculable perturbatively. The resulting predictions are consistent with the empirical facts and electron mass and $\alpha_{U(1)}$ at electron length scale in principle fix the basic parameters of TGD completely.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with n=1 orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Needless to add, if the proposed general picture is correct, not much is left from the super-string/M-theory approach to quantum gravitation since perturbative quantum field theory as the fundamental corner stone must be given up and because the underlying physical picture about gravitational interaction is simply wrong.

3 Victories of M-theory from TGD view point

The basic victories of the M-theory relate to dualities and black hole physics and it is useful perform comparison with TGD.

3.1 Dualities

The starting point of duality physics was the classical paper of Montonen and Olive about electric-magnetic duality [25] which was generalized to what are known as S and T dualities in superstring context. The notion of duality is central also in TGD framework.

3.1.1 Dualities as victories of M-theory

Dualities [26] allowing to unify various superstring models are regarded as basic victories of M-theory. The heuristic proofs for various dualities between various variants of superstring model that I have seen apply what might be called M-logic. Consider special examples defined by 11-dimensional super-gravity using a particular background and particular spontaneous compactification and demonstrate that these examples are consistent with the duality. Then generalize from special to general. For a non-specialist, it is difficult to decide, whether all this is just wishful thinking and clever choices of compactifications.

3.1.2 Mirror symmetry of Calabi-Yau manifolds

String theory has stimulated very general conjectures about the properties of Calabi-Yau manifolds, which have turned out to be correct. Calabi-Yau manifolds are 3-dimensional Kähler manifolds with $SU(3)$ (rather than $U(3)$) holonomy group and thus satisfy empty space Einstein equations implied by the requirement of the vanishing of conformal anomaly in closed super string models. The prediction of the mirror symmetry for Calabi-Yau manifolds [27] emerged before the era of M-theory from the study of $N=2$ super-conformal sigma models with Calabi-Yau manifold as a target space and closed string world sheet as the "space-time". In the 11-dimensional M-theory context Calabi-Yau manifolds are obtained only by a special compactication for which 11^{th} dimension corresponds to a circle. The argument taken from [27] written in a physicist friendly manner runs as follows.

a) In conformal field theories the so called marginal operators correspond to the deformations of the original conformal field theory respecting the property of being a conformal field theory, and thus the criticality of the physical system. In particular, the deformations of complex and Kähler structures of the target space, now Calabi-Yau space, induce this kind of deformations. The basic finding was that the operators inducing these two kinds of deformations differ only by the opposite sign of their U(1) charge associated with the $U(1)$ current of $N=2$ super-symmetry algebra.

b) The mere change of the sign of $U(1)$ charge would correspond to a permutation of the spaces of complex and Kähler moduli which means a rather drastic geometric and even a topological change. On the other hand, the physical change must be marginal since the system remains critical. Both signs of $U(1)$ charge seem highly plausible so that the hypothesis is that the Calabi-Yau manifolds appear a mirror pairs so that in a rough sense the moduli for Kähler and complex structures are permuted for the members of the mirror pair by performing a change of sign of $U(1)$ charge for the left moving modes of string. Actually a generalization of the notion of Kähler moduli is necessary. This is achieved by combining the Kähler form and antisymmetric field B defining a generalization of $U(1)$ gauge potential to form a imaginary and complex parts of a more general structure for which Kähler moduli space (Kähler cone) is complexified and by introducing so called extended Kähler cone combining the Kähler moduli associated with several Calabi-Yau spaces so that single Calabi-Yau manifold can have several mirrors [27].

There are two implications. First, two different Calabi-Yau geometries and even topologies give rise to the same conformally invariant physics: the physics $\leftrightarrow$ geometry identification of General Relativity is not strictly true anymore. Secondly, the continuous change of the complex moduli for the Calabi-Yau manifold corresponds to a topology change for the mirror manifold so that even topology change corresponds to a quite smooth change of physics, in fact a change respecting 2-dimensional criticality. Even the possibility that the change involves a temporary contraction of the Calabi-Yau to a point during the change cannot be excluded [27], which looks really weird. Also singular Calabi-Yau manifolds are possible and not mere limiting cases of non-singular ones [27].

These implications might be also seen as a failure of the theory basically due to the spontaneous compactification trick. In TGD imbedding space is fixed and similar phenomenon does not occur. The moduli space of conformal structures of the metrically 2-dimensional light like CDs effectively corresponding to closed string word sheets is however involved also now, and implies naturally the concept of elementary particle vacuum functional defined in the moduli space of complex structures characterizing the effectively 2-D induced metrics at CDs [F1]. The notion is essential for p-adic mass calculations and predicts correct ratios for electron, muon, and tau lepton masses [F3].

To conclude, the discovery of the mirror symmetry is quite beautiful and impressive but as such does not provide support for the super string theory as a physical theory. The discovery could have been made by a conformal field theorist interested in two-dimensional critical statistical systems.

3.2 Dualities and conformal symmetries in TGD framework

The reason for discussing the rather speculative notion of dualities before considering the definition of the modified Dirac action and discussing the proposal how to define Kähler function in terms of Dirac determinants, is that the duality thinking gives the necessary overall view about the complex situation: even wrong vision is better than no vision at all.

The first candidate for a duality in TGD is electric-magnetic duality appearing in the construction of configuration space geometry. Also the 7-3 duality between 7-D and 3-D CDs relating closely to quantum gravitational holography and YM-gravity duality and representing basically field-particle duality suggests itself. In this case strict duality seems however too strong an assumption.

3.2.1 Electric-magnetic duality

Electric-magnetic duality for the induced Kähler induced field is present also in TGD (CP_2 Kähler form is self-dual). My original belief was that it corresponds to a self duality leaving Kähler coupling constant invariant as an analog of critical temperature: $\alpha_K \rightarrow \alpha_K$ in this transformation [B2, B3]. This duality would allow to construct configuration space Kähler metric in terms of Kähler electric or magnetic fluxes.

It is however possible to imagine a second variant of electric-magnetic duality, not in fact a genuine duality, could correspond to $\alpha_K \rightarrow -\alpha_K$ [B2, B3, E3], which would be a logarithmic version of $g \rightarrow 1/g$ duality and makes sense in TGD framework since g_K does not appear as a coupling constant. This "duality" would have nothing to do with whether the configuration space Hamiltonians are defined in terms of Kähler magnetic or electric fluxes. The two directions of geometric and subjective time and two possible signs of inertial energy would correspond to the two signs (phase conjugate photons would provide example of negative energy particles propagating in the direction of geometric past). Electric *resp.* magnetic flux tubes would be favored in the two phases and a beautiful consistency with TGD inspired cosmology results [D5]. It is questionable whether one can speak of duality now since the two phases seem to be physically different so that alternative descriptions would not be in question. In any case, if a genuine duality is in question it is enough to use either future directed 7-D CDs or past directed CDs. If not, both are required.

This duality relates in an interesting manner to the idea that space-time surfaces can be regarded either hyper-quaternionic sub-manifolds of M^8 endowed with hyper-octonionic tangent space or as 4-surfaces in $M^4 \times CP_2$ [E2]. The point is that one can consider also the dual definition for which the 4-D normal space defines 4-D subalgebra of 8-D algebra at each point of the space-time surface. One might speak of number theoretical spontaneous compactification. Future-past duality could basically reduce to this purely geometric duality and would basically reflect bra-ket duality. The construction of S-matrix however leads to the conclusion that the sign of α_K must be same for for future and past oriented past sheets since otherwise a conflict with unitarity results [C2].

3.2.2 Duality of 3-D and 7-D causal determinants as particle-field duality

As already described, TGD predicts two kinds of super-conformal symmetries corresponding to 7-D and 3-D causal determinants and that their duality would generalize the age-old field-particle particle duality so that quantum gravitational holography and YM-gravitational duality could be seen as particular aspects of field particle duality. The two dual super symmetry algebras defined by super-canonical and Super Kac-Moody algebras at configuration space level define spectrum generating algebras whereas at space-time level they define pure super gauge symmetry algebras eliminating half of the helicities of the induced Dirac spinor fields at each point.

1. The conformal symmetries associated with 7-D CDs and space-time interior

The super-canonical conformal invariance is associated with 7-dimensional light like CDs $\delta M^4_{\pm}(a) \times CP_2$ and their unions at the level of the imbedding space. The GRT counterpart is the moment of local "big bang". The string model counterpart is a Kaluza-Klein type representation of quantum numbers used in superstring models relying on closed strings. Obviously this corresponds to the field aspect of the duality.

At space-time level these dynamical super-conformal symmetries have gauge super-symmetries as their counterpart. The idea of spontaneous number theoretical compactification discussed detail in [E2] allows a more precise formulation of the somehow messy notion of quaternion conformal invariance. Hyper-quaternion analytic maps of the space-time surface regarded as a surface in hyper-octonionic M^8 induce deformations of boundaries of the space-time surface and could represent conformal transformations becoming dynamical at boundaries and causal determinants. These solutions would correspond to pure gauge degrees of freedom in the interior. Also the construction of space-time surfaces in terms of hyper-octonion real-analytic maps of M^8 could be interpreted in terms of a dynamical hyper-octonionic conformal symmetry. These symmetries could be regarded as kind of hidden symmetries in $X^4 \subset M^4 \times CP_2$ picture.

The hyper-quaternion variant of conformal symmetry could be equivalent with $N = 4$ local gauge super-symmetry ($N = 4$ super-symmetric YM theory has been proposed to be closely linked with string models). There would be no global super symmetry. Since the quaternion conformal symmetry can make sense only in interior, and since only the induced spinor field in the interior of space-time surface contributes to the configuration space super charges, quaternion conformal symmetry indeed corresponds to the field aspect of the field-particle duality.

2. Conformal symmetries associated with 3-D light like CDs and quantum gravitational holography

The Super-Kac Moody symmetry at the 3-D light like causal determinants and super-canonical symmetry at 7-D causal determinants define configuration space super-symmetries. These super-symmetries are dynamical and contrary to the original beliefs do not imply the existence of sparticles.

The conformal symmetry associated with the 3-dimensional light like CDs reduces to a generalization of the ordinary super-conformal symmetry. The derivative of the normal coordinate disappears from the modified Dirac operator and solutions are 3-dimensional spinorial shock waves having a very natural interpretation as representations of elementary particles. The GRT analog is black hole horizon. $N=4$ superconformal symmetry in an almost ordinary sense is in question.

The induced spinor fields carry electro-weak quantum numbers as YM type quantum numbers, Poincare and color isometry charges, but not color as spin like degrees of freedom. Hence color degrees of freedom are analogous to rigid body rotational degrees of freedom of the 3-D causal determinant and genuine configuration space degrees of freedom having no counterpart at space-time level although conserved classical color charges make sense: obviously Kaluza-Klein type quantum numbers are in question.

7–3 duality however suggests that it is possible to code the information about configuration space color partial wave to the induced spinor field at X^3_l (for 7-D CDs this is not necessary) as a functional of X^3_l. The guess is that the shock wave solutions of the modified Dirac equation at 3-D CDs can be constructed by taking imbedding space spinor harmonics, operating on them by appropriate color Kac Moody generators to get a correct correlation between electro-weak and color quantum numbers, and applying the modified Dirac operator D to get a spinor basis $D\Psi_m$. If the spinor basis obtained in this manner satisfies $D\Psi_m = c_{mn} o\Psi_n$, where o is the contraction of the light like normal vector of CD with the induced gamma matrices appearing in the eigenvalue equation $D\Psi = \lambda o\Psi$ and defining boundary states for the induced spinor fields and Dirac determinant, the construction works. The fact that quark color does not have a direct space-time counterpart (imbedding space spinors allow color partial waves but induced spinors do not) might correlate with color confinement and with the impossibility to detect free quarks.

3. 7–3 duality and effective 2-dimensionality of 3-surfaces

Whether super-canonical and Kac-Moody algebras are dual is not at all obvious. The assumption that the situation reduces to the intersections X_i^2 of the 3-D CDs X_l^3 with 7-D CDs defining 2-sub-manifolds of X^3 concretizes the idea about duality. Duality would imply effective 2-dimensionality of 3-surfaces and the task is to understand what this could mean.

a) By duality both X_i^2-local H-isometries and the Hamiltonians of $\delta M_+^4 \times CP_2$ restricted to X_i^2 span the tangent space of CH. A highly non-trivial implication would be a dramatic simplification of the construction of the configuration space Hamiltonians, Kähler metric, and gamma matrices since one could just sum only over the flux integrals over the sub-manifolds X_i^2. The best that one might hope is that it is possible to fix both 3-D light like CDs and their sub-manifolds X_i^2 and 7-D CDs freely. There would be good hopes about achieving the p-adicization of the basic definitions.

One could assign unique modular degrees of freedom to X_i^2: this would be crucial for the unique definition of the elementary particle vacuum functionals [F1]. This would give rather good hopes of achieving a better understanding of why particle families corresponding to genera $g > 2$ are effectively absent from the spectrum. Elementary particle vacuum functionals vanish when $g > 2$ is hyper-elliptic, that is allows Z_2 conformal symmetry. The requirement that the tangent space 2-surface defines at each point a commutative sub-space of the octonionic tangent space might force $g > 2$ surfaces to be hyper-elliptic.

b) The reduction to dimension 2 could be understood in terms of the impossibility to choose X^3 freely once light like 3-D CDs are fixed but this does not remove the air of paradox. The resolution of the paradox comes from the following observation. The light likeness condition for 3-D CD can be written in the coordinates for which the induced metric is diagonal as a vanishing of one of the diagonal components of the induced metric, say g_{11}:

$$g_{1i} h_{kl} \partial_1 h^k \partial_i h^l \quad = \quad 0 \ , \ \ i = 1,2,3 \ . \tag{1}$$

The condition $g_{11} = 0$ is exactly like the light likeness condition for the otherwise random M^4 projection of CP_2 type extremals [D1]. When written in terms of the Fourier expansion this conditions gives nothing but classical Virasoro conditions. This analog of the conformal invariance is different from the conformal invariance associated with transversal degrees of freedom and and from super Kac-Moody conformal invariance and its commutative version. This symmetry conforms nicely with the duality idea since also the boundary of the light cone allows conformal invariance in both light like direction and transversal degrees of freedom.

One can consider two interpretations of this symmetry.

i) The degrees of freedom generating different light like 3-D CDs X_l^3 with a given intersection X^2 with 7-D CD correspond to zero modes. Physically this would mean that in each quantum jump a complete localization occurs in these degrees of freedom so that particles behave effectively classically. With this interpretation these degrees of freedom could perhaps be seen as dual for the zero mode degrees of freedom associated with the space-like 3-surfaces X^3 at 7-D CDs: deformation of X_l^3 would induce deformation of X^3.

ii) Gauge degrees of freedom could be in question so that one can make a gauge choice fixing the orbits within certain limits. The two symmetries could correspond to two different choices of gauge reflected as a choice of different space-time sheets. This would mean additional flexibility in the interpretation of this symmetry at the level of solutions of the modified Dirac equation.

At the level of configuration space geometry the result would mean that one can indeed code all data using only two-dimensional surfaces X_i^2 of X^3. This brings in mind a number theoretic realization for the quantum measurement theory. That only mutually commuting observables can be measured simultaneously would correspond to the assumption that all data about configuration space geometry and quantum physics must be given at 2-dimensional surfaces of H for which the tangent space at each point corresponds to an Abelian sub-algebra of octonions. Quantum TGD would reduce to something having very high resemblance with WZW model. One cannot deny the resemblance with M-theories with M interpreted as a membrane.

4. 7–3 duality and the equivalence of loop diagrams with tree diagrams

The 3-D light like CDs are expected to define analogs of Feynman diagrams. In the simplest case there would be past of future and past directed 7-D CDs $X^7_\pm = \delta M^4_\pm \times CP_2$, and the lines of the generalized Feynman diagram would begin from X^7_+ and terminate to X^7_-. In [C5] the generalization of duality symmetry of string models stating that generalized Feynman diagrams with loops are equivalent with tree diagrams is discussed. By quantum-classical correspondence this would mean that the conformal equivalence for Feynman diagrams defined by 3-D light like CDs generalizes to a topological equivalence. This is indeed as it should be since it is the intersections X_i^2 with $X^7_\pm$ which should code for physics and these intersections do not contain information about loops.

Interesting questions relate to the interpretation of the negative energy branches of the space-time surface. It would seem that also the surfaces X^2_i are accompanied by negative energy branch. The branching brings in mind a space-time correlate for bra-ket dichotomy. The two branches would represent Feynman diagrams which are equivalent but correspond to different sign of Kähler coupling strength if the generalization of electric-magnetic duality is accepted.

5. 7-3 duality and quantum measurement theory

The action of Super Kac-Moody generators on configuration space Hamiltonians is well defined and one might hope that as a functional of 2-surface it could give rise to a unique superposition of super-canonical Hamiltonians. Same should apply to the action of super-canonical algebra on Kac Moody algebra. At the level of gamma matrices the question is whether the configuration space metric can be defined equivalently in terms of anti-commutators of super-canonical and Super Kac-Moody generators. If the answer is affirmative, then 7–3 duality would be nothing but a transformation between two preferred coordinates of the configuration space.

TGD inspired quantum measurement theory suggests however that the two super-conformal algebras correspond to each other like classical and quantal degrees of freedom. Super Kac-Moody algebra and super conformal algebra would act as transformations preserving the conformal equivalence class of the partonic 2-surfaces X^2 associated with the maxima of the Kähler function whereas super-canonical algebra in general changes conformal moduli and induces a conformal anomaly in this manner. Hence Kac-Moody algebra seems to act in the zero modes of the configuration space metric. In TGD inspired quantum measurement zero modes correspond to classical non-quantum fluctuating dynamical variables in 1-1 correspondence with quantum fluctuating degrees of freedom like the positions of the pointer of the measurement apparatus with the directions of spin of electron. Hence Kac-Moody algebra would define configuration space coordinates in terms of the map induced by correlation between classical and quantal degrees of freedom induced by entanglement.

Duality would be also realized in a well-defined sense at the level of configuration space conformal symmetries. The idea inspired by Olive-Goddard-Kent coset construction is that the generators of Super Virasoro algebra corresponds to the differences of those associated with Super Kac-Moody and super-canonical algebras. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-canonical local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. If the central extension parameters are same, the resulting central extension is trivial. What is done is to construct first a state with a non-positive conformal weight using super-canonical generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and thus mass.

3.2.3 Quantum gravitational holography

The so called AdS/CFT duality of Maldacena [36] correspondence relates to quantum-gravitational holography states roughly that the gravitational theory in 10-dimensional $AdS_{10-n} \times S^n$ manifold is equivalent with the conformal field theory at the boundary of AdS_D factor, which is $D-1$-dimensional Minkowski space. This duality has been seen as a manifestation of a duality between super-gravity with Kaluza-Klein quantum numbers (closed strings) and super Yang-Mills theories (open strings with quantum numbers at the ends of string).

In TGD quantum gravitational holography is realized in terms of the modified Dirac action at light like 3-D CDs [B4], which by their metric 2-dimensionality allow superconformal invariance and are very much like world sheets of closed super string or the ends of an open string.

It is possible to deduce the values of Kähler action at maximally deterministic regions of space-time sheets from the Dirac determinants at the CDs [B4, E3] so that the enormously difficult solution of the absolute minimization of Kähler action would be reduced to local data stating that CDs are light like 3-surfaces which are also minimal surfaces in the case that Kähler action density is non-vanishing at them. This reduction has enormous importance for the calculability of the theory. Also the values of Kähler coupling strength and gravitational constant are predicted [E3].

Here a word of warning is in order: I do not know how to prove that the minimal surface property of the CDs implies the absolute minimization of Kähler action. One can even consider the possibility that absolute minimization is replaced with the minimal surface property in the case that the action density does not vanish at the CD.

Perhaps the most practical form of the quantum gravitational holography would be that the correlation functions of $N = 4$ super-conformal field theory at the light like 3-D CDs allow to construct the vertices needed to construct S-matrix of quantum TGD. Computationally TGD would reduce to almost string model since light like CD:s are analogous to closed string word sheets on one hand, and to the ends of open string on the other hand.

There is also an analogy with the Wess-Zumino-Witten model: light like CDs would correspond to the 2-D space of WZW model and 4-surface to the associated 3-D space defining the central extension of the Kac-Moody algebra.

Quantum gravitational holography could also mean that light like CDs define what might be called fundamental central nervous systems able to represent and process conscious information about the interior of the space-time surface in terms of its own quantum states which have interpretation either as a time evolution or state (duality again!). Topological quantum computation might be one of the activities associated with the light like CDs as proposed in [O3].

3.3 Number-theoretical spontaneous compactification

The dimensions of space-time and imbedding space suggest that quaternions and octonions should play important role in the formulation of TGD. These ideas have now developed to what might be called number theoretical spontaneous compactification. This approach even suggests that TGD allows a dual formulation as 8-dimensional string theory.

3.3.1 Hyper-quaternions and -octonions

The original idea was that space-time surfaces could be regarded as four-surfaces in 8-D imbedding space with the property that the tangent spaces of these spaces can be locally regarded as 4- *resp.* 8-dimensional number fields of quaternions and octonions.

The difficulties caused by the Euclidian metric signature of the number theoretical norm have however forced to give up the original idea as such, and to introduce complexified octonions and quaternions resulting by extending quaternionic and octonionic algebra by adding imaginary units multiplied with $\sqrt{-1}$. This spoils the number field property but the notion of prime is not lost. The sub-space of hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$. The transition is the number theoretical counterpart of the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity.

The problem is that $H = M^4 \times CP_2$ cannot be endowed with a hyper-octonionic manifold structure. Indeed, space-time surfaces are assumed to be hyper-quaternionic or co-hyper-quaternionic 4-surfaces of 8-dimensional Minkowski space M^8 identifiable as the hyper-octonionic space HO. Since the hyper-quaternionic sub-spaces of HO with fixed complex structure are labelled by CP_2, each (co)-hyper-quaternionic four-surface of HO defines a 4-surface of $M^4 \times CP_2$. One can say that the number-theoretic analog of spontaneous compactification occurs.

3.3.2 Space-time surface as a hyper-quaternionic sub-manifold of hyper-octonionic imbedding space?

Space-time identified as a hyper-quaternionic sub-manifold of the hyper-octonionic space in the sense that the tangent space of the space-time surface defines a hyper-quaternionic sub-algebra of the hyper-octonionic tangent space of H at each space-time point, looks an attractive idea. Second possibility is that the tangent space-algebra of the space-time surface is either associative or co-associative at each point. One can also consider possibility that the dynamics of the space-time surface is determined from the requirement that space-time surface is algebraically closed in the sense that tangent space at each point has this property. Also the possibility that the property in question is associated with the normal space at each point of X^4 can be considered. Some delicacies are caused by the question whether the induced algebra at X^4 is just the hyper-octonionic product or whether the algebra product is projected to the space-time surface. If normal part of the product is projected out the space-time algebra closes automatically.

The first guess would be that space-time surfaces are hyper-quaternionic sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that complex structure is fixed and same at all points of space-time surface. This corresponds to a global selection of a preferred octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2. This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

For a theoretical physicists of my generation it is easy to guess that the next step is to realize that it is possible to fix the preferred octonionic imaginary at each point of HO separately so that local $S^6 = G_2/SU(3)$, or equivalently the local group G_2 subject to $SU(3)$ gauge invariance, characterizes the possible choices of hyper-quaternionic structure with a preferred imaginary unit. $G_2 \subset SO(7)$ is the automorphism group of octonions, and appears also in M-theory. This local choice has interpretation as a fixing of the plane of non-physical polarizations and rise to degeneracy which is a good candidate for the ground state degeneracy caused by the vacuum extremals.

$OH - -M^4 \times CP_2$ duality allows to construct a foliation of HO by hyper-quaternionic space-time surfaces in terms of maps $HO \rightarrow SU(3)$ satisfying certain integrability conditions guaranteing that the distribution of hyper-quaternionic planes integrates to a foliation by 4-surfaces. In fact, the freedom to fix the preferred imaginary unit locally extends the maps to $HO \rightarrow G_2$ reducing to maps $HO \rightarrow SU(3) \times S^6$ in the local trivialization of G_2. This foliation defines a four-parameter family of 4-surfaces in $M^4 \times CP_2$ for each local choice of the preferred imaginary unit. The dual of this foliation defines a 4-parameter famity co-hyper-quaternionic space-time surfaces.

Hyper-octonion analytic functions $HO \rightarrow HO$ with real Taylor coefficients provide a physically motivated ansatz satisfying the integrability conditions. The basic reason is that hyper-octonion analyticity is not plagued by the complications due to non-commutativity and non-associativity. Indeed, this notion results also if the product is Abelianized by assuming that different octonionic imaginary units multiply to zero. A good candidate for the HO dynamics is free massless Dirac action with Weyl condition for an octonion valued spinor field using octonionic representation of gamma matrices and coupled to the G_2 gauge potential defined by the tensor 7×7 tensor product of the imaginary parts of spinor fields.

The basic conjecture is that the absolute minima of Kähler action correspond to the hyper-quaternion analytic surfaces. This conjecture has several variants. It could be that only asymptotic behavior corresponds to hyper-quaternion analytic function but that that hyper-quaternionicity is general property of absolute minima. It could also be that maxima of Kähler function correspond to this kind of 4-surfaces. The encouraging hint is the fact that Hamilton-Jacobi coordinates appear naturally also in the construction of general solutions of field equations.

3.3.3 The notion of Kähler calibration

Calibration is a closed p-form, whose value for a given p-plane is not larger than its volume in the induced metric. What is important that if it is maximum for tangent planes of p-sub-manifold, minimal surface with smallest volume in its homology equivalence class results.

The idea of Kähler calibration is based on a simple observation. The octonionic spinor field defines a map $M^8 \rightarrow H = M^4 \times CP_2$ allowing to induce metric and Kähler form of H to M^8. Also Kähler action is well defined for the local hyper-quaternion plane.

The idea is that the non-closed 4-form associated the wedge product of unit tangent vectors of hyper-quaternionic plane in M^8 and saturating to volume for it becomes closed by multiplication with Kähler action density L_K. If L_K is minimal for hyper-quaternion plane, hyper-quaternionic manifolds define extremals of Kähler action for which the magnitudes of positive and negative contributions to the action are separately minimized.

This variational principle is not equivalent with the absolute minimization of Kähler action. Rather, Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant (they carry non-vanishing density gravitational energy). The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself. The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable construction of Kähler function.

3.3.4 Generalizing the notion of $HO-H$ duality to quantum level

The obvious question is how the $HO-H$ duality could generalize to quantum level. Number theoretical considerations combined with the general vision about generalized Feynman diagrams as a generalization of braid diagrams lead to general formulas for vertices in HO picture.

Simple arguments lead to the conclusion that strict duality can make sense only if the octonionic spinor field is second quantized such that the real Laurent coefficients correspond to a complete set of mutually commuting Hermitian operators having interpretation as observables. Space-time concept is well defined only for the eigen states of these operators and physical states are mapped to space-time surfaces. The Hermitian operators would naturally correspond to the state space spanned by super Kac-Moody and super-canonical algebras, and quantum states would have precise space-time counterparts in accordance with quantum-classical correspondence.

The regions inside which the power series representing HO analytic function and matrix elements of G_2 rotation converge are identified as counterparts of maximal deterministic regions of the space-time surface. The Hermitian operators replacing Laurent coefficients are assumed to commute inside these regions identifiable also as coherence regions for the generalized Schrödinger amplitude represented by the HO spinor field.

By quantum classical correspondence these regions are correlates for the final states of quantum jumps. The space-like 3-D causal determinants X^3 bounding adjacent regions of this kind represent quantum jumps. The octonionic part of the inner of the octonionic spinor fields at the two sides of the discontinuity defined as an integral over X^3 gives (or its sub-manifold) a number identifiable as complex number when imaginary unit is identified appropriately. The inner product is identified as a representation of S-matrix element for an internal transition of particle represented by 3-surface, that is 2-vertex.

For the generalized Feynman diagrams n-vertex corresponds to a fusion of n 4-surfaces along their ends at X^3. 3-vertex can be represented number theoretically as a triality of three hyper-octonion spinors integrated over the 3-surface in question. Higher vertices can be defined as composite functions of triality with a map $(h_1, h_2) \rightarrow \overline{h}_3$ defined by octonionic triality and by duality given by the inner product. More concretely, $m + n$ vertex corresponds in HO picture to the inner product for the local hyper-octonionic products of m outgoing and n incoming hyper-octonionic spinor fields integrated over the 3-surface defining the vertex. Both 2-vertices representing internal transitions and $n > 2$ vertices are completely fixed. This should give some idea about the power of the number theoretical vision.

3.3.5 Does TGD reduce to 8-D WZW string model?

Conservation laws suggests that in the case of non-vacuum extremals the dynamics of the local G_2 automorphism is dictated by field equations of some kind. The experience with WZW model suggests that in the case of non-vacuum extremals G_2 element could be written as a product $g = g_L(h)g_R^{-1}(\overline{h})$ of hyper-octonion analytic and anti-analytic complexified G_2 elements. g would be determined by the data at hyper-complex 2-surface for which the tangent space at a given point is spanned by real unit and preferred hyper-octonionic unit. Also Dirac action would be naturally restricted to this surface. The theory would reduce in HO picture to 8-D WZW string model both classically and quantally since vertices would reduce to integrals over 1-D curves. The interpretation of generalized Feynman diagrams in terms of generalized braid/ribbon diagrams and the unique properties of G_2 provide further support for this picture. In particular, G_2 is the lowest-dimensional Lie group allowing to realize full-powered topological quantum computation based on generalized braid diagrams and using the lowest level k=1 Kac

Moody representation. Even if this reduction would occur only in special cases, such as asymptotic solutions for which Lorentz Kähler force vanishes or maxima of Kähler function, it would mean enormous simplification of the theory.

3.3.6 Precise form of ew-color duality

The e(lectro)w(eak)-color duality associated with $H-HO$ duality reflects the fact that in both pictures the dynamics of single space-time surface can provide only a partial description of quantum dynamics, and that configuration space level is needed in order to code all quantum numbers and all interactions. The situation cries for a more precise formulation for the ew–color duality. The sought for formulation can be expressed as a single concise statement.

In H (HO) picture ew-spin (color) degrees freedom correspond to spin like quantum numbers and color (ew) degrees of freedom to classical conserved charges.

1. *Spin-like quantum numbers and conserved charges in H-picture*

In H picture ew quantum numbers and spin are manifestly present whereas color quantum numbers and interactions emerge as spin like quantum numbers only at configuration space level as does also four-momentum via Kac-Moody representations. Classical color and Poincare charges are well defined also in H picture. There is also a non-trivial interaction between color and ew degrees of freedom since color transformations are accompanied by ew rotations in accordance with the fact that $U(2)_{ew}$ can be mapped to a subgroup of $SU(3)$ via the coset construction.

1. *Spin-like quantum numbers and conserved charges in HO-picture*

Hyper-octonion HO spinors decompose to representations of color group whereas H spinors decompose to the representations of ew and Lorentz group. Hence for HO picture color is manifestly present as spin degrees of freedom but ew spin and spin are absent.

By ew-color duality at space-time level ew and spin charges should somehow emerge also in HO picture as classical conserved quantities. The first observation is that the automorphism group G_2 corresponds to 2 conserved commuting charges. Translating H picture directly to HO level, this would mean that the classical conserved charges associated with WZW + Dirac action have identification as ew charges. Also now a non-trivial relation between electro-weak and color quantum numbers is involved.

There are also symmetries not respecting hyper-octonion real-analyticity and analogous to those affecting the moduli characterizing complex structure. $SO(7)$ leaves the spatial part of the hyper-octonionic norm invariant and this extends the number of conserved charges to 3 bringing in spin. The full isometry group of the hyper-octonionic norm is $SO(7,1)$ so that also Lorentz boost would be included to the Cartan algebra.

Also translations are symmetries of HO picture since the shift of the origin gives rise to a new solution family which is however not hyper-octonion analytic in the original coordinate system. Four- momentum should emerge at quantum level via Kac-Moody type realization also now.

The correspondences $M^4 \times SO(7,1) \leftrightarrow P \times SU(3)$ and $SU(3) \leftrightarrow U(2)_{ew}$ code for the ew–color duality. Interestingly, the four M^4 coordinates depending X^4 coordinates define a local Kac-Moody algebra identifiable in terms of the Cartan algebra of $SO(7,1)$ and extendable by $k = 1$ vertex operator construction to a representation of $SO(7,1)$ Kac-Moody algebra. On the other hand, the Euclidian stringy degrees of freedom in M^4 give rise to $SU(3)$ Kac Moody algebra and to $SU(3)/U(2)$ WZW model serving as a candidate for a model of ew interactions. A very tight web of correspondences between various symmetries is involved.

3. Ew-color duality and and parton-string duality

$G_2/SU(3)$ coset WZW theory would correspond naturally to QCD and geometrically to the identification of a preferred hyper-octonion imaginary unit defining a foliation of HO by string like 2-surfaces. $SU(3)/U(2)$ corresponds naturally to ew gauge theory. These WZW models are obviously mutually exclusive if defined at time-like stringy surfaces Y^2. One can however consider defining $SU(3)/U(2)$ WZW type model at the partonic 2-surfaces X^2 in H picture. These two dual space-time pictures might allow a rather satisfactory quantum-classical correspondence.

That information from entire light-like causal determinants is needed, conforms with the quantum gravitational holography, and the objection that the theory is equivalent to a string model can be avoided. The duality corresponds two kinds of conformal symmetries: hyper-octonionic conformal invariance and the conformal invariance associated with the light-like causal determinants.

There is always room for one counter argument. Parton-string duality does not quite conform with the QCD based notion of parton as a state possessing color as spin like quantum numbers since color is now at configuration space level. This deviation from QCD one must however accept in TGD framework.

4. Ew-color duality and duality of long and short p-adic length scales

The first formulation [F5] for ew-color duality was in terms of p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer, preferably prime or power of prime, as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now.

The proposal was that QCD type description based on quarks and gluons corresponds to a description in the ultra-short length scale L_k and the description in terms of hadrons possessing only electro-weak quantum numbers and spin corresponds to the hadronic length scale L_p. The order of magnitude for α_s is predicted correctly directly from the fact that it is proportional to α_K and as $U(1)$ coupling increases towards short p-adic length scales in a manner predicted by heuristic arguments assuming that gravitational constant does not run appreciably as a function of p-adic length scale.

Indeed, HO picture describing color as a spin like quantum number is more appropriate near CP_2 length scale whereas H picture describing color classically (as in color flux tube models) is more appropriate in hadronic length scales. Perturbative–non-perturbative QCD duality would thus also correspond to $HO - H$ duality. Strictly speaking, non-perturbative QCD would be a meaningless notion.

3.3.7 $H - HO$ duality at the level of configuration space

An interesting challenge is to translate $H - HO$ duality to the level of configuration space geometry and spinor structure.

a) In H picture CH Hamiltonians correspond to Hamiltonians of $\delta M^4_+ \times CP_2$ in representations of $SO(3) \times SU(3)$ whereas spin and electro-weak spin correspond to spin degrees of freedom associated with complexified gamma matrices acting as super-generators.

b) In HO picture CH is replaced with what might be called CHO. The guess is that also CHO allows Kähler and symplectic structures. CHO Hamiltonians cannot correspond to Hamiltonians of E^7 (imaginary hyper-octonions) since E^7 has wrong dimension. 7-D light-cone is in turn metrically a 6-sphere. If S^6 does not allow complex structure as Chern's last theorem claims, it does not allow Kahler structure neither. Situation changes if one considers $\delta M^4_+ \times E^4$ metrically equivalent to $S^2 \times E^4$, which certainly allows Kähler and symplectic structures. This choice is of course perfectly natural and consistent with the view that number theoretical compactification takes effectively E^4 to CP_2 by attaching to it a 2-sphere at infinity. $SO(3)\times SO(4)$ would assign to Hamiltonians spin and ew quantum numbers. Color quantum numbers would correspond to spin degrees of freedom associated with CHO gamma matrices acting also as super generators. $H-HO$ duality could be also interpreted as a super-symmetry permuting bosonic and fermionic degrees of freedom at the level of configuration space.

The obvious question is what is the counterpart of configuration space Kähler function [B1] in HO picture. The identification of CH and CHO Kähler functions is not attractive physically since nothing would happen for the quantum fluctuations. A possible answer comes from the identification of $HO-H$ duality in terms of the purely geometric duality of hyper-quaternionic and co-hyper-quaternionic space-time surfaces. The idea is that it is possible to assign to a given partonic or stringy 2-surface either hyper-quaternionic or co-hyper-quaternionic 4-surface and that these assignments define equivalent theories.

The dual of the Kähler action would be obtained by replacing the induced CP_2 Kähler form $J_{\alpha\beta}$ with its projection J^N_{kl} to the normal space of the space-time surface. This means a contraction with the projector P to the normal space

$$J^N = PJP \ , \ \ P = h - \nabla h\cdot\nabla h \ , \tag{2}$$

where h denotes the imbedding space metric and $\nabla h\cdot\nabla h$ denotes the H-tensor defined by the space-time inner products of gradients of H coordinates h^k. More explicitly,

$$\begin{aligned} J^N_{kl} &= P_k^{\ r}P_l^{\ s}J_{rs} \ , \\ P^{kl} &= h^{kl} - g^{\mu\nu}\partial_\mu h^k\partial_\nu h^l \ . \end{aligned} \tag{3}$$

The dual action would be defined by the dual action density

$$\begin{aligned} L &= k_1 J^N_{kl}J^{kl}_N\sqrt{g} \ , \\ k_1 &= \frac{1}{16\pi\hat{\alpha}_K} \end{aligned} \tag{4}$$

integrated over the space-time surface.

The identification of the dual Kähler coupling strength $\hat{\alpha}_K$ is not quite obvious. The standard form of electric-magnetic duality corresponding to the replacement $g \rightarrow 1/g$ is not sensible in the recent case. This leaves two options.

a) The simplest option would be $\hat{\alpha}_K = \alpha_K$ so that the different dynamics defined by the dual Kähler action would be responsible for the strong–weak duality. This option is not terribly attractive but cannot be excluded.

b) The identification suggested by p-adic length scale hypothesis and $p \leftrightarrow k$ duality of long and short p-adic length scales would be $\hat{\alpha_K}(k) = \alpha_K(p)$ where $p \simeq 2^k$ labels the space-time sheet at which CP_2 type extremals has suffered topological condensation. This identification would reduce the value of coupling strength and thus also the importance of quantum fluctuations in HO picture at short length scale limit. This identification would suggests that CP_2 type extremal *resp.* the space-time sheet of size of order Compton length at which it is topologically condensed are optimally described as co-hyper-quaternionic *resp.* hyper-quaternionic surface and that the transition from the description to its dual corresponds to the exchange of these surfaces.

The possibility of electric-magnetic duality at configuration space level was conjectured already more than decade ago, and was inspired by the observation that configuration space Hamiltonians could be defined in terms of either generalized magnetic or electric fluxes [B2, B3]. This duality could naturally correspond to $HO - H$ duality. Magnetic fluxes are very natural for the flux Hamiltonians defined by the space-like partonic 2-surfaces in H picture, whereas electric fluxes (actually magnetic fluxes for time-like 2-surfaces) are natural for the Hamiltonians associated with time-like string orbits in HO picture. The Kähler functions associated with CH and CHO Kähler metrics defined by these representations would correspond to Kähler function and its dual.

3.3.8 Color confinement and its dual as limits when configuration space degrees of freedom begin to dominate

The description of duality at the configuration space level can be applied to gain a view about color confinement and its dual for electro-weak interactions at short distance limit. The correct prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics.

There are two basic types of vacuum extremals: CP_2 type extremals representing elementary particles and vacuum extremals having CP_2 projection which is at most 2-dimensional Lagrange manifold and representing say hadron. It is not surprising that HO-H duality can be interpreted in terms of these vacuum extremals and they provide a more precise view about what happens at the limits when either CH or CHO degrees of freedom begin to dominate over space-time degrees of freedom describable ordinary quantum field theory.

1. Short distance limit

Consider first the short distance limit at which electro-weak confinement is expected and HO picture becomes more appropriate.

a) Ew-color duality would suggests that at the limit of short distances something analogous to color confinement occurs for electro-weak interactions. Also the large value of $U(1)$ coupling supports this expectation. The vacuum property of CP_2 type extremals means that induced spinor fields become vacuum spinor fields with identically vanishing Dirac action. Therefore these spinor fields effectively disappear at space-time level for the maxima of Kähler function, and contribute only via quantum fluctuations, which correspond to configuration space dynamics. Color partial waves are left as a genuine configuration space degree of freedom and the expectation is that only the lowest color partial waves corresponding to singlet and triplet remain and become spin like degrees of freedom analogous to QCD color in HO picture.

b) Duality suggests $SO(4)$ confinement in E^4 degrees of freedom at this limit. The nearly vacuum property should allow very large fluctuations of the ordinary fermion and anti-fermion numbers at the limit when the fermions become pure vacuons for which creation and annihilation operators reduce to anti-commuting Grassmann numbers. In HO picture this would mean that high $SO(4)$ partial waves in E^4 are possible for composites although net ew quantum numbers vanish. Hence electro-weak spins become analogous to classical angular momentum at this limit.

2. Long distance limit

Consider next color confinement at the long length scale limit as a dual of this picture.

a) In the case of color interactions very high color partial waves for quarks and gluons appear at the confinement limit. For instance, vacuum extremals representable as maps $M^4 \to CP_2$ identifiable as hadronic space-time sheets correspond to color confinement limit. Strong fluctuations due to high color partial waves in CH appear, and correspond in CHO picture to the presence of very high colored hyper-octonionic fermion and anti-fermion numbers. Since configuration space degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD just as high energy limit transcends the descriptive power of standard model of electro-weak interactions.

b) The success of $SO(4)$ sigma model in the description of low lying hadrons could directly relate to the fact that this group labels also the E^4 Hamiltonians in HO picture. $SO(4)$ quantum numbers can be identified as right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks.

c) Pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted. The map of electro-weak spin like degrees of freedom to E^4 degrees of freedom maps quark-anti-quark pairs to E^4 coordinates.

d) Baryons should be analogous to color partial waves of quarks, and just as CP_2 spinors allow at CH level color triplet partial waves also hyper-octonionic fermions should allow at CHO level $SO(4)$ partial waves transforming as doublets under $SU(2)_L$ or $SU(2)_R$.

e) Family replication phenomenon is described in the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [F4].

f) Ordinary fermion numbers do not fluctuate at the color confinement limit. That this does not occur must relate to the facts that modified Dirac action relates by super-symmetry to Kähler action and the variations of Kähler action vanish up to third order around canonically embedded M^4 whereas for the CP_2 type extremals the situation is completely different. The absence of fluctuations in ew spin degrees of freedom suggests the possibility of describing low energy hadrons using simple valence quark model without quark color with quark and gluon sea modelling the presence hyper-octonionic quark pairs. The problems due to statistics might be resolved by anyonic statistic possible for 2-D partonic surfaces. At the asymptotic freedom limit hyper-octonionic sea becomes less and less important.

3.3.9 Proton spin crisis as a signature of hyper-octonionic quarks

Hyper-octonionic quarks carry neither ordinary nor electro-weak spin since these quantum numbers correspond to orbital quantum numbers in HO.

Hence in the ideal colored quark description the contribution of quarks to both spin and electro-weak spin of proton should vanish whereas in H quark description quarks should give proton spin. Obviously, these descriptions correspond to colored current quark description and to a static color singlet quark descriptions possible for anyonic statistics [41, O3]. This prediction would sound crazy unless the essence of proton spin crisis were just the finding that the contribution of quarks to proton spin is small [44, 45, 46, 47].

Spin-statistics paradox is avoided if configuration space degrees of freedom are taken into account. Quantum-classical correspondence, if taken at extreme, would suggest that configuration space degrees of freedom might have some kind of space-time correlate. The 2-dimensionality of stringy and partonic surfaces suggests that anyons might provide this correlate. In HO picture spin-statistics paradox at space-time level would be avoided by the 2-dimensionality of partonic surfaces allowing to have braid representations of the rotation group and colored quarks can have half-odd integer valued anyonic spin and electro-weak spin.

A possible physical mechanism transforming H quarks without color spin but with ew- and ordinary spin to HO quarks having only color spin is following. Anyonic and ordinary contributions to ew- and ordinary spin of H quark cancel each other and color spin is generated anyonically. In TGD framework anyons are associated with punctures assignable to the thin flux threads connecting partonic 2-surfaces and these punctures appear always as pairs with the ends of thread carrying opposite anyonic quantum numbers. OH fermions would correspond to fermion plus the second end of the anyon thread.

In H picture the approach to confinement means large fluctuations also in $SO(3)$ degrees of freedom and the emergence of Regge trajectories. In HO picture the angular momentum of hadron would be due the angular momentum of a large number of colored quark pairs.

3.3.10 Summary

It seems that $HO-H$ duality involves an entire web of dualities suggested by the general structure of TGD. Electric-magnetic duality; duality of hyper-quaternionic and co-hyper-quaternionic 4-surfaces; 7-3 duality stating that either space-like 3-surfaces in the intersections of space-time surface with light-like 7-surfaces $\delta M^4\pm \times CP_2$ or light-like 3-surfaces X_l^3 at which X^4 metric becomes degenerate can be taken as causal determinants, and parton-string duality which string theorist would probably call closed-open string duality, would reduce to the same fundamental duality. These dualities correspond to physical dualities such as ew-color duality described above, $p \simeq 2^k \leftrightarrow k$ duality of long and short p-adic length scales, duality of current quarks and static quarks, and duality of hadron and quark level descriptions.

$HO - H$ duality leads to concrete predictions. Some of them are already verified ($SO(4)$ chiral symmetry of low energy hadron physics and explanation of proton spin crisis). Some of them might be testable (colored quarks should not contribute to right and left electro-weak isospin of hadrons). The most important prediction is that both standard model of electro-weak interactions and QCD are in a precisely defined sense wrong theories. One might hope that this might allow to develop an experimental arrangements proving or disproving this prediction.

Spin-statistics paradox was the crucial observation leading to the introduction of quark color. The 2-dimensionality of partonic and stringy surfaces allowing anyonic statistics, or probably equivalently, the presence of configuration space degrees of freedom, allows to circumvent the spin- statistics paradox otherwise implied by the fact that H spinors do not carry color as spin like quantum number and HO spinors do not carry spin and ew spin.

3.4 Black hole physics

3.4.1 M-theory and black holes

The reproduction of the formula for the black hole entropy [37, 26] has been sold as a victory of M-theory. The first thing that has been forgotten is that GRT based formula has never been experimentally verified and could be even wrong.

One can also criticize the procedure leading to the formula.

First M-theory is replaced by 11-D super gravity in order to calculate something. What this effectively means that, although the aim was to replace General Relativity with something more fundamental, one ends up with 11-D classical super-gravity after all.

After this one finds black-hole type solutions and identifies them with M-branes. At this step one could protest by saying that the fundamental theory should replace black holes with something less singular.

Next quantum gravitational holography is assumed and a conformal field theory on brane identified as a black hole horizon leads to an estimate for the entropy and estimates for what are known as greyness factors. The last step is nice in the 4-D situation and also TGD would suggest something very similar.

In Matrix Theory based estimate things look even less elegant. In [29] a matrix theory based estimate for the entropy is made producing the correct order of magnitude for the entropy estimate using conformal field theory. An essential step is the estimate for the number N of 0-branes (ordinary particles) and is ad hoc (in particular one does not take the limit $N \to \infty$). I do not whether the arguments are more rigorous in other estimates but, to put it mildly, I do not find this argument is not too convincing.

3.4.2 Black holes in TGD framework

Black holes are possible in TGD framework but are basically astrophysical objects and putting black holes and elementary particles in the same basket would be mixing apples with oranges. It however seems that black hole like objects populate TGD Universe in all length scales and are analogous to stringy black holes thought to be tightly tangled strings.

In TGD Universe the role of black hole horizons is taken by various 3-D light like CDs just as the role of big bang is taken by 7-D light like CDs. The basic example is provided by elementary particle horizons surrounding the ends of the wormhole contacts having Euclidian signature of the induced metric and connecting with each other space-time sheets with Minkowskian signature of the induced metric. Second example is provided by light-like surfaces separating maximal deterministic regions of the space-time sheet. Light-like boundaries is a further example. By their metric 2-dimensionality various CDs indeed allow conformal field theory in an effectively 2-dimensional sense. The formula for the black hole entropy generalizes to elementary particle level and involves p-adic length scale hypothesis and p-adic mass calculations [E5].

Further and more concrete ideas about black hole like structures emerged from the attempts to understand the strange events reported by RHIC (Relativistic Heavy Ion Collider) [42, 43] during last years. This work led to a dramatic increase of understanding of TGD and allowed to fuse together separate threads of TGD [D5].

a) The scaled down TGD inspired cosmology involving (not so) big crunch followed by (not so) big bang serves as a model for the events, and predicts a new phase identifiable as color glass condensate identifiable as tightly tangled color magnetic flux tube modellable as a hadronic string in Hagedorn temperature.

This state makes a phase transition to quark gluon plasma during a period of critical cosmology analogous to inflationary cosmology characterized completely by its duration and quark gluon plasma analogous to radiation dominated cosmology in turn hadronizes giving rise to the analog of matter dominated cosmology.

The assumption that conformal confinement is responsible for the formation of the gluonic Bose-Einstein condensate explains also the liquid like character of color glass condensate. Conformal confinement means that individual partons can have complex conformal weights expressible in terms of non-trivial zeros of Riemann Zeta such that the net conformal weight is real. This obviously forces the system to behave like a single particle like unit.

b) RHIC events suggest processes analogous to the formation and evaporation of black hole. The TGD inspired description in terms of the formation of hadronic black hole and its evaporation and essentially identical with the description as a mini bang. The hadronic black hole is the same tightly tangled color magnetic flux tube that defines the initial state of the hadronic mini bang. The attribute 'hadronic' means that Planck length is replaced with hadronic length so that strong gravitation is in question. Black hole temperature is identifiable as Hagedorn temperature and predicted to be 195 MeV for bosonic strings in 4-D space-time and slightly higher than the hadronization temperature measured to be about 176 MeV [D5].

c) Also dark matter could be identified as a macroscopic quantum phase in which individual particles have complex conformal weights. This phase could be even responsible for the properties of living matter [J6]. There is also a connection with the dramatic findings suggesting that Planck constant for dark matter has a gigantic value [**?**, D6].

d) Black holes and their scaled counterparts would not be merciless information destroyers in TGD Universe. The entanglement of particles possessing different conformal weights to give states with a vanishing net conformal weight and having particle like integrity would make black hole like states ideal candidates for quantum computer like systems [O3]. One could even imagine that the galactic black hole is a highly tangled cosmic string in Hagedorn temperature performing quantum computations the complexity of which is totally out of reach of human intellect! Indeed, TGD inspired consciousness predicts that evolution leads to the increase of information and intelligence, and the evolution of stars should not form exception to this. Also the interpretation of black hole as consisting of dark matter follows from this picture [J6].

3.5 Super-conformal symmetries

Space-time super-symmetries are regarded as one of the basic predictions of the super string model. Typically these super-symmetries appear at the level of effective quantum field theory limit derived from spontaneous compactification and predict that massless particles possess massless super partners, sparticles. The problem has been how to generalize Higgs mechanism to break the space-time super-symmetry. That sparticles have relatively low mass scale has been seen as one of the absolute predictions of M-theory and the ability to predict at least something has been counted as a success. Since sparticles have hitherto escaped the attempts to detect them, even this belief has been now challenged, and proposals has been made that perhaps M-theory might after all predict sparticles to be very massive.

Before continuing it must be emphasized that TGD and standard views about super-symmetry differ in many respects.

a) The standard view is inspired by the mathematically awkward and formal idea of assigning to the space-time coordinates anti-commuting super part. The belief is that string world sheet super-symmetries give rise to the space-time super symmetries of the low energy effective quantum field theory assigned to the string model.

b) In TGD the super-symmetry generators of the spectrum generating super-conformal algebra act as gamma matrices of the configuration space ("world of classical worlds"). The counterparts of the word sheet super-symmetries act as gauge super-symmetries at space-time level but do *not* give rise to global space-time super-symmetries at the level of imbedding space. Anti-commuting infinitesimals are encountered nowhere.

3.5.1 Super-symmetry at the space-time level

The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD X^3_l. Gauge symmetries are in question when the intersections of X^3_l with 7-D CDs X^7 are not changed. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced configuration space consisting of the light like 3-D CDs. The conformal symmetries in question imply a further degeneracy of the configuration space metric and effective metric 2-dimensionality of 3-surfaces as a consequence. These conformal symmetries are accompanied by $N = 4$ local super conformal symmetries defined by the solutions of the induced spinor fields.

Contrary to the original beliefs, these conformal symmetries do not seem to be continuable to quaternion conformal super symmetries in the interior of the space-time surface realized as real analytic power series of a quaternionic space-time coordinate. The reason is that these symmetries involve both transversal complex coordinate and light like coordinate as independent variables whereas quaternion conformal symmetries are algebraically one-dimensional.

A resolution of the interpretational problems came with the realization that it is hyper-quaternionic and -octonionic conformal symmetries, which are in question and that these symmetries are naturally associated with the description of the space-time surface as a 4-surface in hyper-quaternionic $HO = M^8$ rather than in H. These symmetries are realized also at the level of H. Note that hyper-quaternionic symmetries act trivially in the interior of X^4 but induce deformations of boundaries of X^4.

The solutions of the modified Dirac equation $D\Psi = 0$, define the modes which do not contribute to the Dirac determinant of the modified Dirac operator in terms of which the vacuum functional assumed to correspond to the exponent of the Kähler action is defined. Thus they define gauge super-symmetries. Usually D selects the physical helicities by the requirement that it annihilates physical states: now the situation is just the opposite. D^2 annihilates the generalized eigen states both at space-like and light like 3-surfaces. Hence the roles of the physical and non-physical helicities are switched. It is the generalized eigen modes of D with non-vanishing eigenvalues λ, which code for the physics whereas the solutions of the modified Dirac equation define super gauge symmetries.

At the space-like 3-surfaces associated with 7-D CDs the spinor harmonics of the configuration space satisfy the $M^4 \times CP_2$ counterpart of the massless Dirac equation so that non-physical helicities are eliminated in the standard sense at the imbedding space level. The righthanded neutrino does not generate an $N = 1$ space-time super-symmetry contrary to the long held belief.

3.5.2 Super-symmetry at the level of configuration space

The gamma matrices of the configuration space are defined as matrix elements of properly chosen operators between right-handed neutrino and second quantized induced spinor field at space-like boundaries X^3 [B4]. These generators define the fermionic generators of what I call super-canonical algebra. The right handed neutrino can be replaced with any spinor harmonic of the imbedding space to obtain an extended super-algebra, which can be used to construct the physical states.

The requirement that super-generators vanish for the vacuum extremals requires that the modified Dirac operator D_+ or the inverse of D_- appearing in the matrix element of the "Hermitian conjugate" $S^- = (S^+)^\dagger$ of the super charge S^+. Here $\pm$ refers to the negative and positive energy space-time sheets meeting at X^3 or to the two maximally deterministic space-time regions separated by the causal determinant. The operators D_+ and D_-^{-1} are restricted to the spinor modes not annihilated by $D_\pm$. The super-generator generated by the covariantly constant right handed neutrino vanishes identically: a more rigorous argument showing that $N = 1$ global super symmetry is indeed absent.

If the configuration space decomposes into a union of sectors labelled by unions of light cones having dips at arbitrary points of M^4, the spinor harmonics can be assumed to define plane waves in M^4 and even possess well-defined four-momenta and mass squared values. Same applies to the super-canonical generators defined by their commutators. This means that the generators of the super-canonical algebra generated in this manner would possess well defined four-momenta and thus their action would change the mass of the state. Space-time super-symmetries would be absent. Similar argument applies to the Kac Moody algebras associated with the light like 3-D CDs if super-canonical Super Kac-Moody algebras provide dual representations of quantum states.

If the gist of these admittedly heuristic arguments is correct, they force to modify drastically the existing view about space-time super-symmetries. The problem how to break super-symmetry disappears since there is no space-time symmetry be broken down. Super-symmetries are realized as a spectrum generating algebra rather than symmetries in the standard sense.

I hasten to admit that I have myself believed that right handed neutrino defines a global super-symmetry and proposed that the topological condensation of sparticles and particles at space-time sheets with different p-adic primes would provide an elegant model for super-symmetry breaking using same general mass formulas but only a different mass scale. Giving up this assumption causes however only a sigh of relief. The predicted spectrum of massless states is reduced dramatically [F3]. p-Adic mass calculations based on p-adic thermodynamics and representations of super-conformal algebra are not affected since the global $N = 1$ super-symmetry implies only an additional vacuum degeneracy. Most predictions of TGD remain intact. The speculation that sneutrinos might be light and play a role in TGD based condensed matter physics is the only possible exception. One can however consider the possibility of light colored sneutrinos obtained by applying to a neutrino state a colored and thus non-vanishing super-canonical generator defined by right handed antineutrino.

It deserves to be noticed that the notion super-symmetry in configuration space sense was discovered with the advent of super string models and generalized to a space-time super-symmetry when gauge theories made their breakthrough. The notion of spontaneous compactification (we meet our friend again and again!) inspired then the hypothesis that this supersymmetry has a space-time counterpart and everyone believed. There is now an entire industry making similar purely formal out of context applications and generalizations of quantum groups, which originally emerged naturally in knot and braid theory and in the theory of von Neumann algebras [E10, O3].

4 A more precise view about $HO-H$ and $HQ-coHQ$ dualities

There are many open questions related to the proposed dualities and the requirement of overall internal consistency and complete symmetry between H and HO pictures gives hopes of achieving a global view about the situation. The understanding of $HO-H$ dualities in terms of momentum and position representations in the cotangent bundle of configuration space reduces the duality to wave-particle duality in infinite-dimensional context. This picture generalizes also to the case of strings and allows to understand what spontaneous compactification means if the notion of stringy configuration space is introduced. $HQ-coHQ$ duality could in turn generalize to duality of branes of dimension d and co-branes of same co-dimension. A detailed discussion of various dualities in TGD is contained in the chapter [E2] of 'TGD'.

4.1 CHO metric and spinor structure

Configuration space metric is defined by the flux Hamiltonian basis of the configuration space in one-one correspondence with Hamiltonians of $\delta M^4_\pm \times CP_2$ in case of CH and $\delta M^4_\pm \times E^4$ in case of CHO [B2, B3]. E^4 has a natural Kähler structure and the most natural assumption is that E^4 Kähler form defines symplectic and Kähler structure of CHO. CHO Hamiltonians would be defined by Hamiltonians in $\delta M^4_\pm \times E^4$ coordinates belonging in irreducible representations of $SO(3,1)\times SO(4)$.

Also CHO should have spinor structure with gamma matrices acting as super generators. Hyper-octonionic spinor fields with real Laurent coefficients cannot be used to construct super generators and second quantization is needed. The only consistent interpretation is that hyper-octonionic spinor fields correspond to zero modes just as solutions of modified Dirac equation correspond to zero modes.

$HO - H$ duality requires that also second quantized HO spinor fields have $1 + 1 + 3 + \overline{3}$ decomposition identifiable naturally in terms leptons, quarks and corresponding anti-fermions. What looks strange is that HO spinor field contains components with both lepton and quark number as well as components with opposite quark/lepton numbers. Conservation laws are however respected since $SU(3)$ does not transform these components to each other. In principle the coefficients of second quantized spinor fields are complex numbers commuting with octonion units.

4.2 Can one interpret $HO - H$ duality and $HQ - coHQ$ duality as generalizations of ordinary q-p duality?

It would be highly desirable to reduce the dualities to familiar notions of mathematical physics so that they could be seen as predictions of TGD rather than hypothesis.

4.2.1 $HO - H$ duality and cotangent bundle of CH

A possible interpretation for $CHO - HO$ duality is in terms of ordinary q-p duality generalized to geometric quantization at the level of the cotangent bundle of the configuration space. The points of the fiber of cotangent bundle would be generalizations of canonical momenta. What would be *new* that these canonical momenta would be representable as 4-surfaces in HO, and obtained by assigning to each point of 4-surface of H a co-tangent vector in the fiber of cotangent bundle of H.

The infinite-dimensional generalization of wave-particle duality would allow to use either the base space CH or CHO defining the fiber of cotangent bundle or more generally, Lagrange manifold of the cotangent bundle. If this interpretation is correct, any Lagrange manifold of cotangent bundle is a priori admissible apart from constraints posed by infinite-dimensional existence. Symmetries of course pose natural constraints to the Lagrange manifolds.

Furthermore, covariant gamma matrices are in one-one correspondence with the cotangent space basis so that the permutation of spin and orbital degrees of freedom is consistent with q-p permutation.

$HQ - coHQ$ duality brings in mind the generalization of ordinary $q - p$ duality associated with cotangent bundle to a duality defined by the normal bundle of X^4. This requires that space-time dimension is half of the dimension of imbedding space. $HQ - coHQ$ duality would thus permute base and fiber of the normal bundle. What is new that points in the fiber of the normal bundle would have representation as 4-surfaces of H. This is possible if the normal of 4-surface X^4 is identified as a map assigning to each point of X^4 a vector in normal space mapped to point of H somehow so that these points define a surface of H. If the CP_2 part of the normal vector is interpreted as a vector in the complement of $U(2)$ Lie-algebra and exponentiated and projected back to CP_2 a surface in H results.

4.2.2 What is the physical interpretation of $HQ - coHQ$ Fourier transform?

Fourier transform maps functions in base space of cotangent bundle to functions in the fiber of cotangent bundle and both $HO - H$ and $HQ - coHQ$ dualities should correspond to a generalized Fourier transform.

An interesting question is whether $HQ - coHQ$ Fourier transform might have some deeper interpretation. Consider a given HQ 4-surface X^4 and its dual determined by Bohr quantization mediating $HQ - coHQ$ duality. These surfaces have a discrete set of common points in the generic case and one might wonder whether this set of points could have interpretation as punctures representing states created by completely localized (topological?) quantum fields in X^4. The simplest situation is that there exist for each point of X^4 single $coHQ$ surface going through it and this surface intersects X^4 in single point. The questions are following.

a) Is it possible to interpret elements of second quantized Fourier basis in X^4 in terms of functions in the space of 4-surfaces dual to X^4 in this case?

b) What is the interpretation when multiple intersections are possible: do these represent internal degrees of freedom?

c) Could this interpretation give a geometric meaning for n-point functions of quantum fields?

For the foliation of X^4 by stringy 2-surfaces labelled by their partonic duals intersections appear, and the interpretation of the intersection points as punctures representing completely localized states created by conformal or topological quantum field is rather attractive. Also a connection with the 2+2 intersection form of 4-dimensional cohomology playing a key role in the classification of 4-manifolds is suggestive. If this were the case, the intersection form would allow to assign (topological) particle number to space-time surface (or space-time sheet).

4.3 Further implications of $HO - H$ duality

The improved understanding of $HO - H$ duality leads to additional highly non-trivial conjectures.

4.3.1 Does the same 4-D points set represent both q and $p(q)$?

$HO - H$ duality interpreted in terms of cotangent bundle of configuration space forces to ask whether one should not replace Kähler action for $M^4 \times CP_2$ (4-surfaces representing "positions") with its counterpart for $M^4 \times E^4$ in HO picture (4-surfaces representing "momenta"). This would mean a map assigning to a given 4-surface in H its dual in HO. Kind of Bohr quantization would be in question: to a given position of particle (4-surface in H) a unique momentum (4-surface in HO would be assigned.

The idea that these two different 4-surfaces should to different point sets in HO and H does not seem attractive, and the question is whether the same point-set X^4 in HO represents both CH point and CHO point (position q and momentum $p(q)$) and the differences come only from the fact that the metric and Kähler form are induced from HO and H respectively. This would be of course in spirit with the notion of $HO - H$ duality and provide it with an additional beauty and meaning. Also Lagrange manifolds applied in geometric quantization assign unique momentum to given position and thus perform Bohr orbit quantization.

The duality in this strong sense raises several interesting questions.

a) Do vacuum extremals for HO Kähler action co-incide set-theoretically with those for H Kähler action?

b) Is the value of Kähler function defined by the two Kähler actions same for a given 3-surface?

These two requirements imply that all 4-surfaces $X^4 \subset HO$ approach to vacuum extremals at large distances from the origin and asymptotically have at most 2-D E^4 and CP_2 projections. These surfaces need not of course have infinite size in HO.

Infinite primes have interpretation as Fock states and representation as 4-surfaces [E3]. The hyper-octonionic building bricks of infinite primes allow a straightforward interpretation as components of a quantized 8-momentum for a particle in HO and thus correspond to the momentum representation of the theory.

4.3.2 Do both HO and H spinor fields define foliations?

Complete $HO-H$ duality suggests that hyper-octonionic spinor fields should have analogs at the level of H. Solutions of massless Dirac equation for H-spinors are the only reasonable candidates for the counterparts of H spinor fields. These spinor fields should define in some natural manner a map assigning to a point of H a point of HO. The natural guess is that M^4 coordinates result by canonical projection whereas E^4 coordinates e^k correspond to the currents $e^k = \overline{\Psi}\Gamma^k_{CP_2}\Psi$. Ψ is fixed apart from a local $U(1)$ phase and the consistency condition would be that Ψ (which have either quark or leptonic chirality) satisfies massless Dirac equation in H. If a superposition of quark and lepton like currents is allowed, the changes of satisfying the condition are better. The Kähler calibration for HO Kähler action should define same 4-surface in set theoretic sense as that for H Kähler action.

4.3.3 Are induced spinor fields restrictions of imbedding space spinor fields?

As already found, the strict $HO - H$ duality supports the view that the induction to X^4 makes sense for both H and HO spinor fields, that the zero modes of these spinor fields satisfying massless Dirac equation at the level of H (HO) and modified Dirac equation at the level of $X^4 \subset H$ ($X^4 \subset HO$) correspond to classical degrees of freedom, and that second quantized part defines the gamma matrices of configuration space CH (CHO) acting as super-generators.

The introduction of imbedding space and space-time spinor fields as independent dynamical degrees of freedom does not look a good idea and the simplest assumption is that the allowed induced spinor fields or at least the zero modes for these spinor fields in the foliation correspond to the restrictions of H and HO spinor fields. This would select only single zero mode from the space of all allowed ones.

For the second quantized X^4 spinor fields the representability as a restriction of second quantized H spinor field would mean that the anti-commutators of H spinor fields are non-vanishing at surfaces X^4 rather than at 7-D surfaces as in ordinary quantization. This would resolve the longstanding problem how to perform induction procedure for quantized H spinor fields without producing fatal 7-D delta function singularities to the space-time integrals of anti-commutators of the induced spinor fields.

If the Laurent coefficients of the zero mode spinor fields are Hermitian operators representing observables characterizing quantum states this selection would realize quantum-classical correspondence. Space-time surfaces would serve as representations of quantum states. This assumption is however not absolutely essential and the question of how this correspondence is realized remains unanswered.

4.4 Do induced spinor fields define foliation of space-time surface by 2-surfaces?

$HO-H$ duality in its strongest form would mean a foliation of HO/H by HQ 4-surfaces with the surfaces of the foliation parameterized by the points of any $coHQ$ surface. This foliation is defined by HO spinor field.

One can ask whether the zero modes of induced H spinor field in X^4 could define a foliation of 4-surface. For HQ 4-surface regarded as surface in H this foliation would be by hyper-complex (HC) stringy 2-surfaces labelled by the points of any $coHC$ partonic surface. For $coHQ$ 4-surface both 2-surfaces and co-2-surfaces would be coC surfaces.

How this foliation could be defined by a zero mode of induced H or HO spinor field? The following argument translates the construction in HO case to HQ case.

a) The first guess is that the map $HO \rightarrow M^4 \times CP_2$ generalizes to a map $X^4 \rightarrow M^2 \times S^2$. This means that zero mode must assign to a given point of X^4 points of M^2 and S^2. This map in turn would define a foliation of X^4 by HC 2-surfaces labelled points of any $coHC$ 2-surface so that string model like structure would result.

b) Point of M^2 could be assigned to a point X^4 by selecting a preferred subspace of M^4 an applying canonical projection from X^4 to M^2. Lorentz invariance is a possible source of troubles.

c) One must consider the situation in both HO and H pictures.

i) The hyper-octonionic spinor field mode of HO induced to space-time surface would has with respect to the quaternionic automorphism group $SO(3)$ $1+3$ decomposition.

ii) For a given zero mode of H spinor field with given chirality could be regarded as a 2-component spinor field having two complexified quaternionic components and obeying Weyl condition. If one allows the representation in which quaternionic units are expressed in terms of Pauli sigma matrices, it decomposes into two doublets under the automorphism group $SU(2)$ of complexified quaternions. Also an additional $U(1)$ factor appears naturally from complexification. The identification as representations of electro-weak $U(2)$ would be indeed very natural. If quaternionic units are representable as in terms of induced H gamma matrices this decomposition is natural.

d) Assume a selection of preferred hyper-quaternionic imaginary unit. The automorphisms leaving this unit invariant correspond to the rotation group $SO(2)$ and the possible selections of this unit are labelled by points of S^2 just like in the case of HO the selections remain invariant under $SU(3)$ and are labelled by S^6. S^2 degree of freedom corresponds to a freedom to perform a local rotation for the imaginary part of the argument in hyper-quaternionic power series. That the groups $U(1)$ and $SU(3)$ are exact symmetries of standard model might relate to the foliations in some deep manner. In H picture $U(1) \times U(1)$ defines the invariance group and gives $S^2 = U(2)/U(1) \times U(1)$.

e) In HO picture the tensor product $3 \otimes 3$ for triplet part of Ψ contains triplet identifiable as $SO(3)$ Lie algebra element which can be exponentiated so that it assigns a point of S^2 just as HO spinor field defines point of CP_2. Also the doublet decomposition natural in H picture allows this kind of map. The point of S^2 defines a hyper-complex plane at each point of X^4. Integrable distribution of these planes possibly defined by the analog of Kähler calibration would define a foliation of X^4 by 2-surfaces and its dual.

f) WZW action would presumably emerge as the counterpart of Kähler action now and would define $SO(3)/SO(2)$ coset model in HO picture. In H picture $SO(3))$ is replaced by $U(2)$ and $U(2)/U(1) \times U(1)$ coset model results. These models must be equivalent. $U(2)$ would be identifiable as electro-weak gauge group so that this step in reduction would correspond to electro-weak symmetry breaking. Since quaternions are complexified the $U(1)$ factor emerges naturally.

There are obvious generalizations.

a) The construction should generalize also to *coHQ* surfaces. In this case, the dual 2-surfaces would be naturally complex both.

b) Similar foliation by 2-surfaces and their co-2-surfaces should appear also in HO picture and for the induced spinors obtained from those of HO satisfying the modified Dirac equation defined for the counterpart of Kähler action in HO. A further question inspired by Bohr quantization and duality considerations is whether these foliations are identical set-theoretically and the differences come only from the induce metric and Kähler structure.

4.5 Could configuration space cotangent bundle allow to understand M-theory dualities at a deeper level?

It is interesting to see how TGD dualities relate to U, T and S dualities of M-theory. T duality relates large and small scales, S duality $g \leftrightarrow 1/g$ relates coupling constants and U dualities correspond to products of S and T dualities. Obviously $HQ-coHQ$ duality acts like T that it relates long and short length scales (M^4 type and CP_2 vacuum extremals in extreme situation). In the case of $HQ-coHQ$ duality the Z_2 symmetry permutes minimum and maximum of L_K and tangential and normal degrees of freedom. In the case of $HO-H$ duality spin and orbital degrees of freedom are permuted by Z^2 symmetry. In cotangent bundle picture fiber and base degrees of freedom are exchanged. If the coupling constants associated with k and $p \simeq 2^k$ are mapped to each other, also S duality aspect is involved.

$HO-H$ duality and the generalization of configuration space of 3-surfaces to that of configuration space of 1-surfaces with Kähler metric (which need not exist mathematically except in very rare special cases) allows also a deeper understanding of spontaneous compactification and the dualities of M-theory.

Stringy compactification could be understood in terms of the cotangent bundle of stringy configuration space of strings and its dual defining the representations of canonical momenta and positions in the configuration spaces of strings associated with non-compactified and compactified target space respectively. This would explain why a given flat space theory allows several compactifications. The non-compactified theory would correspond to the perturbative theory which is always the same whereas compactifications would correspond to non-perturbative theories, which are the "real" theories. The duality mapping strings in two representations to each other would be analogous to Bohr quantization assigning to a given position q canonical momentum p(q), and Lagrange manifolds of cotangent bundle of compactified theory would determine a huge variety of different dualities unless there are some constraints from symmetries. Probably these constraints are very important, at least in TGD.

$HQ-coHQ$ duality could generalize at brane level to the duality of branes of dimension d and co-branes of same codimension.

The mysterious dualities would thus reduce to a rather familiar notion of cotangent bundle, the representation of points of the fiber of this bundle as 2-surfaces, and the realization of Bohr orbitology using a generalization of Lagrange manifold allowing to map points of base-space to those of fiber.

Of course, this formal picture need not make sense since the existence of infinite-dimensional Kähler geometry poses so strong constraints that the notion of dynamical target space must be given up since for an arbitrary target space configuration space geometry need not exist at all. This would of course mean getting rid of the landscape problem. Unfortunately, it could also mean that the physics predicted by string models and M-theory does not have much to do with what we see in laboratory.

For instance, the Kähler geometry of loop groups is unique and has Kac-Moody algebras as isometry algebras. This would suggest that the most general choice for target space is as a product of loop groups containing also Abelian factors. Perhaps also loop analogs of products of coset spaces are possible. 2-dimensional general coordinate invariance gives strong constraints and Wess-Zumino-Witten action seems to be a natural candidate for the bosonic part of string action. TGD suggests that strings in 4-D space is the only possible option since it allows the analog of HQ-coHQ duality as an additional symmetry. This would however trivialize the theory since internal symmetries would be absent.

5 What went wrong with string models?

As will be found, the few physical predictions of M-theory are wrong. It is instructive to try to understand what went wrong with M-theories and string models by comparing it with earlier successful theories and with TGD.

5.1 Problems of M-theory

At the physical side the situation in M-theory can be regarded as a catastrophe and without the association of the attribute "the only known candidate for the quantum theory of gravitation..." to the letter M bringing in mind Pavlov dogs, no-one could take it seriously. The various problems of M-theory have been discussed in the article of Smolin [33] as also by Penrose in his lecture series "Fashion, Faith and Fantasy in Theoretical Physics" [30]. The discussions of "Not Even Wrong" [35] group provide a vivid critical view about the situation.

a) M-theory has not been able to explain why the dimension of the space-time is four and has even failed to reproduce the standard model. Unless one assumes that the small dimensions form a singular manifold (something so ugly that it turns my stomach around), M-theory predicts chiral symmetry just like Kaluza-Klein theories: the symmetry is inconsistent with the standard model. Ironically, just this was the reason why superstrings replaced Kaluza-Klein theories in the first superstring revolution. This full π twist represents a good example of M-logic.

The predicted massless scalar fields have not been observed. The predicted low energy super-symmetry is experimentally absent, and now papers have begun to appear suggesting that M-theory after all might predict only high energy super-symmetry. One of the first findings after the second superstring revolution was that the prediction for the unification scale was wrong.

I remember that Witten proposed at that time a suitable compactification of the 11^{th} dimension to a circle to circumvent this problem.

b) Cosmological constant is now believed to be non-vanishing and positive [39] whereas the cosmological constant predicted by M-theory is negative. M-theories provide no explanation for the accelerated expansion [39]. There is a plethora of cosmological observations which M-theory cannot even address.

This sad state of affairs has led to the introduction of the anthropic principle [32] but not in the sense that it would really predict something but as an M-logic proof that M-theory after all predicts among other things also the cosmological constant correctly. The premise is that M-theory is correct and the conclusion is that the observed universe must represent some distant corner of the M-landscape, and we must be ready to accept as a fact, that we will never be able to find our way to this distant part of the Theory Universe, and be happy with learning new dualities.

5.2 Mouse as a tailor

The history of string models differs dramatically from that for theories which has been successful as physical theories. As a rule, new theories have started from a precise problem which earlier theories have not been able to solve, and have led to a new ontology and inspired new mathematics.

String model was born as a model of hadrons. It however became gradually clear that the constraints on space-time dimensions make it unrealistic for this purpose. The conclusion of the mouse was not so humble as in the tale: admittedly string models fail for hadrons but who knows, they might describe everything.

After a decade of tailoring the cat was told that superstrings do not seem to make a TOE after all. The mouse said that he could tailor even something more grandiose just by sewing together all the previous failures. Now it has become clear that the result is an enormous bundle of solutions of the possibly existing M-theory, which at practical level is reduced after few heuristic arguments to compactifications of 11-D super gravity. There is still however a little problem: not a single one of these solutions seems to describe the Universe we live in. Now the mouse suggests that we should give up the dream about a theory of the observable universe as unrealistic, stop complaining and be happy with all these beautiful dualities.

Is the time ripe for the story to end as its original version did or shall the cat provide still another decade of financial support for the expensive tailor?

5.3 The dogma of reductionism

5.3.1 M-theory as an outcome of hard-nosed reductionism

The philosophical background of string models is hard-nosed reductionism taken down to Planck length: something taken to be so self-evident that it has not been even mentioned. Hence the theory cannot make any predictions about or utilize the rich experimental input coming from the known physics.

This means that string theorists do not pay any attention to the pressing problems of quantum measurement theory, to the problems related to the relationship between experienced and geometric time, and to the problems surrounding to the poor understanding of second law. Not to even mention the questions about the difference between animate and in-animate matter, and about what it means to be a conscious system.

The belief that the action defining functional integral summarizes the physics leads to an approach which is extremely pragmatic: start from the existing formulas of perturbative field theories and try to combine them in order to cook up a more general theory. The danger that theoreticians fall into a kind of mathematical insanity in this kind of situation is obvious, and the possible failure of reductionism means a tragic failure of the entire approach.

5.3.2 Giving up reductionism

TGD cannot be regarded as a success from the point of view of sociology of science but the success of TGD as a physical theory is undeniable and basically due to the facts that TGD emerged as a solution to a well-defined problem, and that the notion of many-sheeted space-time plus p-adic length scale hypothesis [E5] provide a precise quantitative formulation for how reductionism fails.

a) I ended up with TGD by starting from a very real problem of general relativity and soon found that I could end up to TGD also from string models. From the beginning the contact of TGD with experimental physics was very intimate. Later the quantum classical correspondence has become a basic guide line in the construction of the theory.

b) One cannot deny that string theories partially solved the divergence problem of perturbative quantum field theories. Unfortunately, is is highly implausible that the sum of the perturbation series would converge so that as such it is useless. This has in fact been seen as a victory of the theory since one can hope that a genuinely non-perturbative approach could lead to a unique theory.

In TGD framework the absence of the basic divergences is highly plausible already from the basic construction involving new ontology of space-time. Vacuum functional identified as an exponent of Kähler function is not anymore a local functional of 3-surface so that basic perturbative divergences resulting from the micro-locality are absent. Also Gaussian and metric determinants cancel and the definition of Kähler function in terms of Dirac determinant is free of divergencies [B4]. The equivalence of generalized Feynman diagrams containing loops with tree diagrams expresses quantum criticality and generalizes the duality symmetry of string models [C5]. The principle can be formulated Hopf algebraically, and provides a genuinely non-perturbative formulation of the theory.

c) The construction of quantum TGD was not possible without the theory of consciousness. Key element is the replacement of space-time micro-locality with classical locality in the "world of classical worlds" making possible to understand how macroscopic and macro-temporal quantum coherence are possible [K6, K2, K4]. Thanks to the notion of self [H3, E7, K1], observer ceases to be an outsider and quantum measurement theory is becomes an essential part of the theory. Completely un-expected outcomes were the already mentioned generalizations of the number concept and the identification of the space-time correlates of cognition and intentionality.

d) TGD generalizes in a dramatic manner the ontology of space-time in terms of the notion of the many-sheeted space-time involving also the new view about numbers. The identification of space-time sheets as space-time counterparts of physical objects resolves the question about the generation of structures. The ontology of quantum TGD is discussed in [E7] from the point of view of category theory. One important implication is that even quantum superposition and quantum logic can have space-time correlates at the level of many-sheeted space-time.

e) TGD resolves the paradoxes due to the conflict between the non-determinism of quantum jump and determinism of Schrödinger equation and, by the classical non-determinism, quantum-classical correspondence can be realized at the space-time level even for quantum jump sequences. TGD leads to a new view about the relationship between geometric and subjectively experienced time rather than just identifying them [K1].

f) In a dual manner TGD makes a distinction between inertial and gravitational masses: quite generally, gravitational quantum numbers are differences for those associated with positive and negative energy matter. The prediction is that Equivalence Principle in its standard form holds true only when the interaction between positive and negative energy matter can be neglected [D3, D5]. Thus the conservation of the inertial momenta is consistent with the non-conservation of gravitational momenta. This should have been obvious from the beginning from the fact that Robertson-Walker cosmologies correspond to vacuum extremals in TGD [D5]: it however took 25 years to realize that Einstein was probably wrong. Implications are rather dramatic. For instance, Universe as a whole has vanishing inertial quantum numbers and is created again in every quantum jump in 4-D sense so that the question about what were the initial values at the moment of big bang becomes obsolete [D5].

Negative energies make possible what I call remote metabolism playing in key role in TGD inspired theory of consciousness and of quantum biology: the system can gain energy by sending negative energy to geometric past [K6, K2, K1]. Time mirror mechanism makes possible communications with geometric past and future and communications with an effectively superluminal velocity become possible.

g) The duality between theory and reality is resolved. TGD based ontology postulates only three levels of existence corresponding to existences in these sense of classical and quantum physics, and conscious existence which corresponds to the quantum jumps between the quantum states [E7]. The possibility that space-time points are infinitely structured in p-adic sense although this structure is not visible in real sense [E10], would resolve the challenge posed by the question why all those structures that we can imagine mathematically, are not realized physically. Obviously, a reincarnation of the monad idea of Leibniz is in question.

5.4 The loosely defined M

In a sharp contrast with M-theory [28], Newton's mechanics and gravitational theory, Maxwell's electrodynamics, Special and General Relativities, and even Bohr's rules were from the beginning relatively precisely defined theories able to make testable predictions. The lack of a precise definition of what "M" means has led to a flood of speculations based on speculations based on...

"M" as "membrane" would be a rather precise definition but does not really make sense since the huge conformal invariance of string models is lost as objects become 2-dimensional. For this reason one prefers to replace "M" with Mystery, Mother, or perhaps Matrix, but still think in terms of membranes which behave like strings. It became however clear that also branes of various dimensions are needed as discovered by Polchinski [38] and identified as non-perturbative objects at which string ends are attached to: this interpretation is the only possible one since otherwise momentum conservation would be lost for D-branes.

Needless to say, a theory using geometric structures consisting of parts possessing different dimensions does not satisfy the standards of the conventional mathematical aesthetics. An outsider could argue that the non-uniqueness of the boundary conditions (Neumann, Dirichlet and mixtures of them) is the fundamental failure of the string theory, and that a viable theory should predict the dynamics of boundaries. This is indeed the case in TGD where absolute minimization of the Kähler action guaranteing general coordinate invariance in 4-D sense does this and implies that the space-time surface is a field theory counterpart of Bohr orbit.

A good example of brave new M-logic is provided by the construction of what is called Matrix Theory [29]. One starts from M-theory "known" to have 11-D supergravity as a low energy limit, replaces it with a 11-D supergravity, restricts the consideration to N 0-branes (point particles) living in an effectively 10-D space, in an ad hoc manner replaces their position coordinates in 10-D space with non-commuting $N\times N$-matrix valued coordinates assuming that eigenvalues correspond to N space-time points, postulates a non-relativistic Schrödinger equation for this matrix, and by generalizing bravely the notion of holography, concludes that the original theory and even more follows from this very-very special theory at $N\rightarrow\infty$ limit. From Matrix Theory one then deduces all superstring dualities and and black hole physics using an argumentation with a comparable rigor.

It must be added that TGD predicts a rich variety of objects resulting as asymptotic self-organization patterns for which Kähler-Lorentz 4-force vanishes by quantum classical correspondence. The solutions are classified by the dimension of either their M^4 or CP_2 projection [D1]. This variety includes cosmic strings and magnetic flux tubes besides space-time sheets. Magnetic flux tubes and string like objects can indeed attach to the boundaries of space-time sheets and there are obvious correspondences with branes with dimensions of branes restricted to run from 0 to 4 ($p=-1,..,3$) but only as objects obtained by idealizing 4-dimensional object with a lower-dimensional object.

Even the possibility of single space-time point or space-time curve to mimic the quantum dynamics of the quantum state of Universe is predicted but only at the level of cognition and relying on the new notion about what mathematical point is [E10]. I however do not think that this has much to do with Matrix Theory.

5.5 Los Alamos, M-theory, and TGD

String models have been seen not only as a kind of holy grail of modern physics but also as an ideology promising an Utopia. As a rule, ideologies have tried to establish the new world order using censorship. String model hegemony has followed the tradition.

For about decade ago it became impossible for me to get anything to hep-th and other physics related archives. Interestingly, for few years ago my article about Riemann hypothesis was accepted to the math archives of Los Alamos and is also published [16]: it was however not possible to get it cross-listed to hep-th. For a few years American Mathematical Society has had a link to my homepage [18] as one of the few examples about new mathematics related to quantum physics.

I have learned that I am not the only victim of the string revolution (see the comments in "Not Even Wrong" discussion group [35]). Despite the official statement that anyone can contribute to LANL, an invisible peer system is acting. After 20 years of string revolutions it seems that physics itself has become the victim which has suffered the most severe injuries.

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